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NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

A Comparative Accuracy of Several Discrete Methods for Lower Confidence Limit on System Reliability

by

Hariono

September 1977

Thesis Advisor: W. M. Woods

Approved for public release; distribution unlimited

Thesis
H228

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The systems simulated had reliabilities ranging from 0.720 to 0.950. They were composed of five, ten, thirteen, and fifteen components, and had component sample sizes of fifteen, thirty, fifty, and larger in the case of unequal sample sizes.

Based on the simulation results the accuracy of the procedures were compared by common comparison with the true system reliabilities which were known in advance prior to the component tests.

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A Comparative Accuracy of Several Discrete Methods for Lower Confidence Limit on System Reliability

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This thesis is a comparative accuracy study of several discrete methods for lower confidence limits on series system reliability. Computer simulations were used to compare the accuracy of the procedures. Five hundred replications were used in all simulations. Accuracy of each procedure was determined by computing appropriate percentile points of the distributions of the lower confidence limits. A randomization technique was used to improve the performance of one of the procedures.

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I. INTRODUCTION

Suppose a complex mechanism, e.g., a missile, is built from a number of different types of components, where the reliability of each of the components has been estimated by means of separate tests on each of the components. There exist many procedures for combining such component data to determine approximate lower confidence limits for the reliability of the system. Several such procedures are the Maximum Likelihood (Ref. 12), the Madansky (Ref. 5), the Log-Gamma (Ref. 13), the Easterling/Modified Maximum Likelihood (Ref. 2) and the Mann (Ref. 6) methods.

This study is concerned with series system. Lower confidence limit accuracies are compared by means of computer simulation. Two methods of introducing partial component failures were used in the simulation to modify some of the procedures which cannot be used when all the components exhibit no failures. The first method was proposed by W. M. Woods and the second by Lisowsky (Ref. 4) who also developed a successful technique for computing the Lagrange multiplier in the Madansky procedure. An attempt was also made to improve the performance of the Easterling method using randomization techniques developed by D. R. Barr and T. Jayachandran (Ref. 1).

The simulation results show that the accuracy of the Maximum Likelihood, Madansky and Mann procedures are comparable, the Log-Gamma and Easterling procedures yield satisfactory results when sample sizes are large and unequal, and the Randomized Easterling procedure yields better results than the Easterling procedure in all cases.

Finally, it is worthwhile to note that the Maximum Likelihood procedure is simple and easy to implement; therefore it can be used as a rough and ready method.

II. DESCRIPTION OF THE METHODS

Consider a system which consists of k components in logical series; the components may be either continuously operating or of the cycle type.

Suppose n_i copies of component i are put on test, i = 1, 2,...,k, under the environmental conditions defined in its mission profile, and let each operate until failure or the mission time is reached, whichever occurs first. Denote f_i as the number of components of type i that did not complete the mission, and define

$$\hat{p}_i = 1 - \hat{q}_i \qquad (2.1)$$

where

$$\hat{q}_i = f_i/n_i$$

The following methods were modified in Chapter III, section C, since some of the procedures cannot be used when all components exhibit no failure, i.e., the Madansky, the Easterling and the Randomized Easterling procedures.

A. THE MAXIMUM LIKELIHOOD (ML) METHOD

The maximum likelihood estimate for system reliability is

$$\hat{R}_s = \prod_{i=1}^{k} \hat{p}_i \qquad (2.2)$$

This estimator is asymptotically normal in distribution, and its variance is estimated by (Ref. 9):

$$\hat{\sigma}^2 = \hat{R}_s^2 \sum_{i=1}^{k} \frac{(n_i - x_i)}{n_i x_i} \qquad (2.3)$$

where $x_i = n_i - f_i$

The ML 100(1-α)% Lower Confidence Limit (LCL) for system reliability is given by

$$\hat{R}_{s,L(\alpha)} = \hat{R}_s - z_{1-\alpha}\hat{\sigma} \qquad (2.4)$$

where $z_{1-\alpha}$ is the 100(1-α) percent point of the standard normal distribution.

B. THE MADANSKY (MD) METHOD

The Madansky method is based on the well-known results due to Wilks (Ref. 11) that $-2 \ln\rho$ is distributed asymptotically as a chi-square random variable with one degree of freedom.

ρ is the likelihood ratio test statistic given by (Ref. 5).

$$\rho = \frac{\left[\max \prod_{i=1}^{k} \mathrm{BIN}(x_i;p_i) \,\middle|\, \prod_{i=1}^{k} p_i = R_s\right]}{\left[\max \prod_{i=1}^{k} \mathrm{BIN}(x_i;p_i)\right]} \qquad (2.5)$$

where $\mathrm{BIN}\ (x_i;p_i) = \binom{n_i}{x_i} p_i^{x_i} (1-p_i)^{n_i - x_i}$

$$i = 1,2,\ldots,k.$$

The numerator is maximized under the additional constraint

$$\prod_{i=1}^{k} p_i = R_s, \text{ and}$$

the denominator is an unconstrained maximization. Values of R_s included in the two sided confidence interval (the confidence set) are given by

$$s(R_s) = [R_s: - 2 \ln \rho \leq \chi^2_{\alpha,1}] \qquad (2.6)$$

where $\chi^2_{\alpha,1}$ is the upper percent point of the chi-square distribution with 1 degree of freedom.

It is easy to see that the logarithm of the denominator of ρ is

$$\sum_{i=1}^{k} \ln \mathrm{BIN}(x_i; \frac{x_i}{n_i}) = \sum_{i=1}^{k} \ln \binom{n_i}{x_i} + \sum_{i=1}^{k} x_i \ln x_i - \sum_{i=1}^{k} x_i \ln n_i$$

$$+ \sum_{i=1}^{k} (n_i - x_i) \ln (n_i - x_i) - \sum_{i=1}^{k} (n_i - x_i) \ln n_i$$

To determine the logarithm of the numerator of ρ, let us first maximize the Lagrangian

$$\sum_{i=1}^{k} \ln \mathrm{BIN}(x_i; p_i) - \lambda\{\ln \prod_{i=1}^{k} p_i - \ln R_s\}$$

where λ is a Lagrange multiplier. The maximizing set of p_i's is given by

$$\hat{p}_i = \frac{x_i - \lambda}{n_i - \lambda},$$

where, since $0 \leq p_i \leq 1$ for all i, then $\lambda \leq \min x_i$.

Hence, as a function of λ,

$$\sum_{i=1}^{k} \ln \mathrm{BIN}(x_i;\, p_i) = \sum_{i=1}^{k} \binom{n_i}{x_i}$$

$$+ \sum_{i=1}^{k} x_i \ln(x_i-\lambda) - \sum_{i=1}^{k} x_i \ln(n_i-\lambda)$$

$$+ \sum_{i=1}^{k} (n_i-x_i)\ln(n_i-x_i) - \sum_{i=1}^{k} (n_i-x_i) \ln(n_i-\lambda)$$

and

$$\ln\rho = \sum_{i=1}^{k} x_i \ln\left(1 - \frac{\lambda}{x_i}\right) - \sum_{i=1}^{k} n_i \ln\left(1 - \frac{\lambda}{n_i}\right), \qquad (2.7)$$

where λ satisfies the constraint equation

$$\prod_{i=1}^{k} \frac{(x_i-\lambda)}{(n_i-\lambda)} = R_s \qquad (2.8)$$

The set of λ such that $-2 \ln \rho \leq \chi^2_{\alpha,1}$ will be an interval $[\lambda_1^*, \lambda_2^*]$, where $\lambda_1^* < 0 < \lambda_2^*$ and the λ^* satisfy

$$\sum_{i=1}^{k} x_i \ln\left(1 - \frac{\lambda}{x_i}\right) - \sum_{i=1}^{k} n_i \ln\left(1 - \frac{\lambda}{n_i}\right) = -\frac{\chi^2_{\alpha,1}}{2} \qquad (2.9)$$

The MD 100(1-α)% LCL is given by

$$\hat{R}_{s,L(\alpha)} = R_s(\lambda_2^*) \qquad (2.10)$$

In determining lower confidence limits the convention adopted by Myhre and Saunders (Ref. 12) will be used where the value of α in equation 2.9 is

$$\alpha = 2(1-\gamma),$$

and γ is the confidence coefficient.

C. THE LOG-GAMMA (LG) METHOD

In the LG procedure the method of moments is used to fit the random variable $-\ln R_s$ with the two parameter gamma distribution (Ref. 13). The gamma is then transformed into a chi-square distribution, about which probability statements are made and a lower confidence limit obtained. That is define

$$S = -\ln R_s = - \sum_{i=1}^{k} \ln(1-q_i) \tag{2.11}$$

Expanding the natural logarithm in an infinite series

$$S = - \sum_{i=1}^{k} [(-q_i) - \tfrac{1}{2} (-q_i)^2 + \tfrac{1}{3} \cdots]$$

and if each q_i is small, the above series can be approximated by the first two terms of the infinite series. That is

$$S = \sum_{i=1}^{k} [q_i + q_i^2/2] = \sum_{i=1}^{k} T_i \tag{2.12}$$

where

$$T_i = q_i + \frac{q_i^2}{2}$$

It has been shown that the error due to the above truncation is negligible in cases of practical interest.

An unbiased estimator $\hat{T}_i$ for T_i (Ref. 7) is

$$\hat{T}_i = a_i \hat{q}_i + b_i \frac{\hat{q}_i^2}{2} \tag{2.13}$$

where $$a_i = \frac{2n_i - 3}{2(n_i - 1)} \tag{2.14}$$

$$b_i = \frac{n_i}{n_i - 1} \tag{2.15}$$

That $\hat{T}_i$ is unbiased is important, because

$$\hat{S} = \sum_{i=1}^{k} \hat{T}_i \tag{2.16}$$

is used as an estimator for S, thereby accumulating any bias present in the T_i. An approximate value for the variance of S (Ref. 7) is

$$\text{Var}\,(\hat{S}) = \sum_{i=1}^{k} \text{Var}\,(\hat{T}_i) \equiv \sum_{i=1}^{k} \frac{\hat{T}_i}{n_i} \tag{2.17}$$

Next, fit $\hat{S}$ with a gamma distribution. The probability distribution of $\hat{S}$ is then given by the density function

$$f_s\,(x;\,r,\,\theta) = \frac{1}{\Gamma(r)\,\theta^r}\,x^{r-1}\,\exp\,(-x/\theta) \tag{2.18}$$

$$x>0,\; r>0,\; \theta>0$$

It follows that

$$E\,(\hat{S}) = r\theta \tag{2.19}$$

$$\text{Var}\,(\hat{S}) = r\,\theta^2 \tag{2.20}$$

Since $\hat{S}$ is unbiased

$$E\,(\hat{S}) = S$$

$$= \sum_{i=1}^{k} T_i \tag{2.21}$$

Solving equations (2.17), (2.19), 2.20) and (2.21) simultaneously gives the shape parameter

$$r = \left[\sum_{i=1}^{k} T_i\right]^2 \Bigg/ \sum_{i=1}^{k} \frac{T_i}{n_i} \tag{2.22}$$

and the scale parameter

$$\theta = \left[\sum_{i=1}^{k} \frac{T_i}{n_i}\right] \Bigg/ \sum_{i=1}^{k} T_i \tag{2.23}$$

Thus, r can be estimated by

$$\hat{r} = \left[\sum_{i=1}^{k} \hat{T}_i\right]^2 \Bigg/ \sum_{i=1}^{k} \frac{\hat{T}_i}{n_i} \tag{2.24}$$

Since $\hat{S}$ is distributed gamma (r,θ), $2\hat{S}/\theta$ is distributed χ^2_{2r}. Then $\chi^2_{1-\alpha,2r}$ is that number such that

$$1 - \alpha = P\,[\chi^2_{2r} \geq \chi^2_{1-\alpha,2r}] \tag{2.25}$$

And since $\hat{S}$ is unbiased

$$\begin{aligned} E(\hat{S}) &= S \\ &= r\theta \\ &= -\ln R_s \end{aligned} \tag{2.26}$$

Equation 2.25 becomes

$$1 - \alpha = P\,[2r\hat{S} \geq -\ln R_s\ \chi^2_{1-\alpha,2r}]$$

$$1 - \alpha = P\left[\exp\left\{\frac{-2r\hat{S}}{\chi^2_{1-\alpha,2r}}\right\} \leq R_s\right] \tag{2.27}$$

Therefore, the LG 100(1-α)% LCL for system reliability R_s is

$$\hat{R}_{s,L(\alpha)} = \exp\left[\frac{-2\hat{r}\hat{S}}{\chi^2_{1-\alpha,2r}}\right] \tag{2.28}$$

To preclude usage of non-integer degrees of freedom, the approximation

$$\frac{[2r\hat{S}]}{\chi^2_{1-\alpha,[2r]}} \tag{2.29}$$

is used in equation (2.28), where $[2\hat{r}]$ denotes the smallest integer greater than or equal to $2\hat{r}$. Approximation (2.29) was shown to have little effect on the LG procedure accuracy (Ref. 7, p. 16). Thus, the 100 (1-α)% LCL becomes

$$\hat{R}_{s,L(\alpha)} = \exp\left[\frac{-[2\hat{r}]\hat{S}}{\chi^2_{1-\alpha,[2\hat{r}]}}\right] \tag{2.30}$$

A continuity correction is used in the LG procedure to improve the accuracy of the lower confidence limit. The reason for the inaccuracy of this lower confidence limit lies in the fact that a continuous distribution has been fitted to $-\ln R_s$. This type of correction has a smoothing effect upon the probability distribution of $\hat{R}_{s,L(\alpha)}$.

The continuity correction is made as follows:

1. Determine that component i_o which has the largest sample size; i.e., $n_{i_o} \geq n_i$, $i = 1, \ldots, k$.

2. Define $\hat{T}_i'$ as

$$\hat{T}_{i_o}' = a_{i_o} \frac{(f_{i_o} + 1)}{n_{i_o}} + \tfrac{1}{2} b_{i_o} \frac{(f_{i_o} + 1)^2}{n_{i_o}}$$

This means that one more failure is added to that component with largest sample size to obtain T_i'

3. Define $\hat{T}_{i_o}^*$ by

$$\hat{T}_{i_o}^* = \tfrac{1}{2} (\hat{T}_{i_o}' + \hat{T}_{i_o})$$

4. Substitute $\hat{T}_{i_o}^*$ for $\hat{T}_{i_o}$ in $\hat{S}$ to obtain $\hat{S}^*$ The resulting $\hat{S}^*$is the continuity corrected value of $\hat{S}$.

5. $\hat{r}$ is corrected to obtain $\hat{r}^*$ by substituting $\hat{T}_{i_o}^*$ for $\hat{T}_{i_o}$ in the definition of $\hat{r}$.

6. With these definitions of $\hat{S}^*$ and $\hat{r}^*$ the new 100 $(1-\alpha)\%$ lower confidence limit $\hat{R}_{s,L(\alpha)}^*$ for R_s becomes

$$\hat{R}_{s,L(\alpha)}^* = \exp \left\{ \frac{-[2\hat{r}^*]S^*}{\chi^2_{1-\alpha,[2\hat{r}^*]}} \right\} \tag{2.31}$$

D. THE EASTERLING/MODIFIED MAXIMUM LIKELIHOOD (MML) METHOD

In this method the ML estimate $\hat{R}_s$ is treated as the usual binomial estimate based on "$\hat{n}$", called the pseudo sample size, is unknown and is estimated from

$$\hat{\sigma}^2 = \frac{\hat{R}_s(1 - \hat{R}_s)}{\hat{n}} \qquad (2.32)$$

where $\hat{\sigma}^2$ is given by 2.3. Thus by equating the estimated variance of $\hat{R}_s$ under maximum likelihood theory to what it would be under binomial theory, we can solve this equation for $\hat{n}$. Then the component test results can be regarded as being equivalent to system results of $\hat{n}$ tests with $\hat{x} = \hat{R}_s.\hat{n}$ successes.

In binomial sampling with $\hat{x}$ successes in $\hat{n}$ trials, a lower $100(1 - \alpha)\%$ confidence limit on the reliability is given by the solution for R_s in

$$I(R_s, \hat{x}, \hat{n} - \hat{x} + 1) = \alpha \qquad (2.33)$$

where I is the incomplete beta function with

$\hat{x}$: the first parameter, and

$\hat{n} - \hat{x} + 1$: the second parameter.

$\hat{n}$ and $\hat{x}$ are unlikely to be integers and the calculations of lower limits in the comparative examples (chapter IV) use Easterling's MML method in which $\hat{n}$ and $\hat{x}$ were rounded up to the next integers.

E. THE RANDOMIZED EASTERLING (RE) METHOD

Barr and Jayachandran in Ref. 1 develop a randomization technique for improving the lower confidence limit on the reliability of a system with a discrete distribution. Basically the method is a simple one, for example an exact $100(1 - \alpha)\%$ LCL for the parameter of the binomial distribution can be obtained as follows:

1. Let $z=x+y$, where y represents an observed value of Y, and Y is uniformly distributed between zero and one inclusive; x is the number of successes in n trials.

2. The solution for R_s in

$$\mathcal{I}(R_s, z, n - z + 1) = \alpha \qquad (2.34)$$

is the exact $100(1-\alpha)\%$ LCL.

Since the Easterling method leads to one component system, equation 2.34 is readily applicable for computing an exact $100(1-\alpha)\%$ LCL based on the Easterling method by replacing z with $\hat{z}$ in equation 2.35, where

$$\hat{z} = \hat{x} + y$$

and n with $\hat{n}$, $\hat{x}$ and $\hat{n}$ as defined in equation 2.33.

F. THE MANN (MN) METHOD

Mann et al. in Ref. 6 shows that the $100(1-\alpha)\%$ LCL on R_s can be obtained by fitting the posterior distribution of $-\ln R_s$ with a noncentral chi-square distribution. The corresponding central chi-square variate with non-integer degrees of freedom is transformed to normality yielding:

$$P\left[R_s \geq \exp\left[-m\left[1-v/(9m^2) + z_{1-\alpha}v^{\frac{1}{2}}/(3m)\right]^3\right]\right] = 1-\alpha \qquad (2.35)$$

where m is the mean and v the variance of the posterior distribution of $-\ln R_s$.

$$m = \frac{0.5(1+1/a)}{n^*} + \frac{1-R_s}{0.5(R_s+1)} \qquad (2.36)$$

$$v = \frac{0.5(1+1/a)}{n^*} \cdot m \qquad (2.37)$$

with

$$a = n_{(1)} \sum_{i=1}^{k} (1/n_i), \qquad (2.38)$$

$$n_{(1)} = \min(n_1, \ldots, n_k), \qquad (2.39)$$

$$n^* = n_{(1)}[1-0.5(1-R_s)^2][1-0.5(1-R_s)], \qquad (2.40)$$

and $z_{1-\alpha}$ = $100(1-\alpha)$-th percentile of the standard normal distribution.

All zero - failure components are ignored in calculating the value of a in equation 2.38 except for any single zero - failure component with its sample size equal to $n_{(1)}$. It, too, is ignored, however, if at least one other component that exhibits failure has sample size equal to $n_{(1)}$.

III. THE SIMULATION

A. GENERAL METHODOLOGY

A computer simulation is used in this study as an analytical tool for evaluating a proposed LCL procedure; the method is as follows:

1. Suppose it is desired to evaluate a proposed $100(1-\alpha)\%$ LCL procedure, denoted $\hat{R}_{s,L(\alpha)}$, for system reliability R_s. Then the assertion is

$$P[\hat{R}_{s,L(\alpha)} \leq R_s] \geq 1-\alpha \tag{3.1}$$

Equality should hold if $\hat{R}_{s,L(\alpha)}$ is a continuous random variable.

2. $R_s = f(p_1,p_2,\ldots,p_k)$, where p_i is the true reliability of the component. For an independent series system, $R_s = \prod_{i=1}^{k} p_i$.

3. Assign values to the parameters α, k, n_i and p_i, $i=1,2,\ldots,k$. Perform n_i Bernoulli trials for the ith component to get the number of successes/failures for each component and then compute the resultant $\hat{R}_{s,L(\alpha)}$.

4. Generate the approximate distribution of $\hat{R}_{s,L(\alpha)}$ by repating step 3 500 times.

5. Order the $\hat{R}_{s,L(\alpha)}$ realizations to get $\hat{R}_{s,L(\alpha)_{(1)}},\ldots,\hat{R}_{s,L(\alpha)_{(500)}}$.

6. Find the $500(1-\alpha)\underline{th}$ order statistic of $\hat{R}_{s,L(\alpha)}$ and denote it $A_{1-\alpha}$. Thus $A_{1-\alpha}$ is the $(1-\alpha)\underline{th}$ percentile of the distribution of the LCL random variable $\hat{R}_{s,L(\alpha)}$.

7. By repeating steps 3 through 6 for various sets of $k, n_1, n_2, \ldots, n_k, p_1, p_2, \ldots, p_k$ and comparing the resultant $A_{1-\alpha}$ to R_s, the overall performance of the LCL procedure can be evaluated.

The actual confidence level given by $\hat{R}_{s,L(\alpha)}$ can be obtained by finding the order statistics of the generated distribution which matches (or is closest to matching) R_s. If the index of this order statistic is denoted i^*, then

$$\frac{i^*}{500} \times 100\% = \text{Actual level of confidence of } \hat{R}_{s,L(\alpha)} \tag{3.2}$$

Equivalently, if $A_{1-\alpha} < R_s$, the procedure is a conservative one, and vice-versa.

B. ACCURACY CRITERIA

There are three characteristics of the distribution of $\hat{R}_{s,L(\alpha)}$ for determining the accuracy of LCL procedure. They are:

1. The mean
2. The variance, and
3. $A_{1-\alpha}$.

The variance should be small and the actual values should still be within the ball park when the LCL is applied.

If in fact, $\hat{R}_{s,L(\alpha)}$ is an exact $100(1-\alpha)\%$ LCL procedure for R_s, that is

$$P[\hat{R}_{s,L(\alpha)} \leq R_s] = 1-\alpha \tag{3.3}$$

then $A_{1-\alpha}$ should be close to R_s regardless of the set of parameter values used and for each value of α.

Thus the quantity

$$\left|A_{1-\alpha} - R_s\right| \tag{3.4}$$

is a measure of the accuracy of the procedure.

C. SIMULATION ALGORITHM

Step 1. Given a set of parameters k, n_i and p_i, $i = 1,2,\ldots,k$, generate binomial data as follows:

For the ith component, $i = 1,2,\ldots,k$, draw a uniformly distributed random number u from the interval [0,1] and compute the number of successes x_i

$$s_i = \begin{cases} 1 & \text{if } u \leq p_i \\ 0 & \text{if } u > p_i \end{cases}$$

$$x_i = \sum_{1}^{n_i} s_i, \quad i = 1,2,\ldots,k. \tag{3.5}$$

All the procedures described in chapter II simply ignore the zero-failure component(s) with the result either the computed LCL tends to be very high or the procedures cannot be applied.

For example if $x_i = n_i$ for all i, then σ^2 in equation 2.3 is zero, and hence the ML's LCL in equation 2.4 is equal to one and the Easterling and Randomized Easterling procedures cannot be applied because $\hat{n}$, the pseudo sample size in equation 2.32, is not defined. Likewise the Madansky procedure cannot be applied (equation 2.9 is not defined). In this case the Log-Gamma's LCL and the Mann's LCL are close to one. To cope with this problem two methods of introducing partial component failures will be presented in this study for those methods which accommodate zero failures.

FIRST METHOD. If $x_i=n_i$ for all i, pick the component with the largest sample size or pick the first component if all sample sizes are equal and introduce a half failure to that component; this method was proposed by W. M. Woods.

SECOND METHOD. Whenever $f_i=0$, set $f_i=4/(n_i k)$, this method was proposed by Lisowsky (Ref. 4).

Step 2. For each α ($\alpha=0.1$ and $\alpha=0.2$) compute the LCL for all procedures as described in chapter II.

Step 3. For each set of data repeat step 1 and step 2 500 times, order the LCL's, and then compute $A_{1-\alpha}$, mean, standard deviation of the LCL distribution, the actual level of confidence in equation 3.2, and the average failure per replica.

Now determine each procedure's accuracy by comparing the $A_{1-\alpha}$'s with the true system reliability R_s, and relative dispersion by comparing their mean and standard deviation.

For detailed computation algorithm see Appendix, The Computer Program.

IV. SIMULATION RESULTS AND CONCLUSIONS

A. GENERAL DESCRIPTION

The accuracy of the procedures are compared for a variety of sets of parameter values (k, p_i, n_i, $i=1,2,...,k$). These different sets of parameters are called cases and are numbered. For each case the two methods of introducing partial component failures were applied at two different confidence levels: 90% and 80% CL.

The simulation results were tabulated in Table II at the end of this chapter; for each case the results were listed in one table, for example Table II.1 for case number 1, Table II.2 for case number 2, etc., up to Table II.36. The letter suffix attached to the case number denotes the method of introducing partial component failures. In particular case 1a means the FIRST METHOD of introducing partial component failures as described on page 24 was applied to case 1. Similarly case 1b means the SECOND METHOD of introducing partial component failures as described on page 24 was applied to case 1.

By varying n_i while k and p_i are held constant in each case, the sensitivity of the procedures can be examined.

B. THE EFFECT OF SAMPLE SIZE

Observing the simulation results in Table II starting from sample size 15 through 50 on each case, these results indicate that increasing sample size (continued on page 27)

TABLE I

NOMENCLATURE FOR SIMULATION RESULTS

k	Number of series connected components
n_i	Sample size of ith component
p_i	True reliability of ith component
R_s	True independent series system reliability, $R_s = \prod_{i=1}^{k} p_i$
m	Sample mean of $\hat{R}_{s,L(\alpha)}$
s	Sample standard deviation of $\hat{R}_{s,L(\alpha)}$
Fbar	Average failure per replica $(1/500) \sum_{1}^{500} \sum_{i=1}^{k} \sum_{1}^{n_i} f_i$
Nfail	Total number of FIRST METHOD of introducing partial component failures applied
Ncorr	Total number of SECOND METHOD of introducing partial component failures applied
CL	$100(1-\alpha)\%$ confidence level
ACL	Actual confidence level, computed using equation 3.2, page 22, the result is rounded to the smallest integer
$A_{1-\alpha}$	The $500(1-\alpha)$th order statistic of $\hat{R}_{s,L(\alpha)}$
ML	Maximum likelihood method
MD	Madansky method
LG	Log-Gamma method
MML	Easterling method
RE	Randomized Easterling method
MN	Mann method

causes the $A_{1-\alpha}$'s to converge to R_s, thus decreasing the value of $|A_{1-\alpha}-R_s|$ of all procedures. The results also indicate that some of the procedures need larger sample sizes to converge satisfactorily.

With medium sample size (30) and large sample size (50) the performances of ML, MD and MN procedures are comparable, although at sample size 30 in some cases MD and MN procedures perform better than ML procedure as shown in case 5, 14 and 23. But with sample size 15 and with $R_s > 0.900$ the ML procedure seems to perform better than MD and MN procedures as shown in cases 1, 16 and 19.

The LG, MML and RE procedures converge rather slowly, but RE tends to converge faster than MML in all cases; this is due to the randomization effect as described in chapter II section E page 19. On the other hand with unequal sample sizes (mix of large, medium and small sample sizes) these procedures tend to converge satisfactorily as shown in case 34, 35 and 36, while the MN procedure always yields unsatisfactory results, and again ML and MD procedures are still comparable under this condition. Note that in these cases only the SECOND METHOD of introducing partial component failures can be applied in the MD procedure as was pointed out by Lisowsky (Ref. 4).

C. THE EFFECT OF THE METHOD OF INTRODUCING PARTIAL COMPONENT FAILURES

When the cases under study have large sample sizes if it can be expected that all components of these cases exhibit failure(s) or only a few components exhibit no failure, then the two methods practically have no effect on the computed LCL's as demonstrated by simulation results using sample sizes of 50 in Table II. These facts were also shown in cases where $p_i < 0.960$ for all i with sample size 30, i.e., case 8, 11 and 14, Table II. The explanation is as follows. Take as an example case 8a where Nfail = 2; this means that the FIRST METHOD of introducing partial component failures was applied twice. Its contribution to the average failure per replica (Fbar = 5.6) was very small: (2)(0.5)(1/500) = 0.002. In case 8b Ncorr = 768, the SECOND METHOD was applied 768 times and its contribution of failure to Fbar (5.7) was {(768x4)/(5x30)}(1/500) = 0.04. Both of these contributions were small compared to Fbar so that the computed $A_{1-\alpha}$'s in case 8a and 8b were approximately the same. Note that the value of Fbar is rounded to the first decimal in Table II.

On the other hand if n_i is small and p_i is high for all i as in cases 1, 16 and 19, then different values of $A_{1-\alpha}$ can be expected from the two methods. For example case 1a, Nfail = 251 and the contribution of failure to Fbar (1.0) was: (251)(0.5)(1/500) = 0.25. In case 1b Ncorr = 2171, its contribution to Fbar (0.9) was {(2171x4)/(5x15)} (1/500) = 0.23. Both of these contributions were quite high compared with Fbar and therefore the computed $A_{1-\alpha}$'s were different, except for MML and RE procedures which were due to the fact that $\hat{n}$ and $\hat{x}$ in chapter II section D and E on pages 18 and 19 rounded up to the next integer. In these cases the ML, MD and MN procedures yield higher and the LG procedure yields lower $A_{1-\alpha}$'s with the SECOND METHOD, but this was not so for the other cases.

A more detailed quantitative analysis is needed to determine the range of values of k, n_i and p_i for which the FIRST METHOD is better than the SECOND METHOD for each procedure and vice versa. This is beyond the scope of this study.

D. CONCLUSIONS AND REMARKS

The overall performance of ML, MD and MN procedures are comparable although with medium sample size (thirty) MD and MN procedures perform better than the ML procedure, but with small sample size (fifteen) the ML procedure seems to perform better than MD and MN procedures.

The LG, MML and RE procedures tend to yield satisfactory results using unequal sample sizes (mix of large, medium and small sample sizes), while the MN procedure always yields unsatisfactory results. The ML and MD procedures are still comparable under these conditions.

The randomization technique was successful since RE procedure yielded better results than MML procedure.

The ML procedure is simple and easy to implement, therefore this procedure can also be used as a rough and ready method. The MD procedure requires a computer to work with while the MN procedure can be solved with a handheld calculator although it is a rather complicated computation.

TABLE II.1

TABULATED SIMULATION RESULTS

Case 1a: $k=5$, $p_i=0.990$, $n_i=15$, $i=1,2,...,5$. $R_s=0.951$

FIRST METHOD: Fbar = 1.0, Nfail = 251

	********90% CL**********				*********80% CL*********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.907	.861	.060	50	.928	.887	.054	50
MD	.872	.830	.056	50	.912	.874	.052	50
LG	.708	.660	.053	50	.818	.776	.049	50
MML	.778	.734	.053	50	.824	.782	.051	50
RE	.847	.773	.062	100	.873	.818	.059	100
MN	.810	.777	.042	50	.855	.823	.041	50

Case 1b: same as 1a

SECOND METHOD: Fbar = 0.9, Ncorr = 2171

ML	.939	.869	.079	50	.954	.893	.070	50
MD	.901	.837	.073	50	.938	.879	.067	50
LG	.652	.627	.020	82	.797	.766	.039	50
MML	.778	.728	.056	50	.824	.777	.054	50
RE	.847	.768	.065	100	.873	.814	.061	100
MN	.885	.829	.065	50	.914	.862	.061	50

TABLE II.2 (Continued)

Case 2a: Same as 1a, except $n_i=30$, $i=1,2,\ldots,5$

FIRST METHOD: Fbar = 1.6, Nfail = 121

	********90% CL**********				********80% CL*********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.953	.899	.050	76	.964	.915	.045	45
MD	.934	.882	.048	76	.956	.908	.044	76
LG	.842	.791	.036	76	.905	.857	.038	76
MML	.880	.824	.045	76	.906	.854	.042	76
RE	.905	.845	.048	100	.913	.873	.045	100
MN	.901	.861	.036	76	.925	.887	.035	76

Case 2b: Same as 2a

SECOND METHOD: Fbar = 1.6, Ncorr = 1844

ML	.980	.902	.058	76	.985	.918	.051	45
MD	.959	.885	.055	76	.977	.911	.050	51
LG	.819	.797	.029	76	.910	.862	.040	76
MML	.880	.823	.044	76	.906	.853	.042	76
RE	.905	.843	.048	100	.913	.872	.045	100
MN	.948	.884	.050	76	.962	.904	.046	76

TABLE II.3 (Continued)

Case 3a: Same as 1a, except $n_i=50$, $i=1,2,...,5$

FIRST METHOD: Fbar = 2.5, Nfail = 49

	*******90% CL**********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.955	.915	.040	68	.963	.928	.037	68
MD	.944	.905	.039	86	.959	.924	.036	68
LG	.893	.860	.032	90	.930	.896	.033	90
MML	.901	.868	.036	90	.920	.889	.034	90
RE	.926	.880	.038	98	.931	.900	.035	93
MN	.925	.897	.031	90	.941	.914	.031	90

Case 3b: Same as 3a

SECOND METHOD: Fbar = 2.5, Ncorr = 1529

ML	.953	.916	.043	68	.962	.928	.039	68
MD	.942	.906	.042	72	.957	.924	.038	68
LG	.889	.860	.030	90	.927	.897	.034	90
MML	.899	.868	.036	90	.918	.888	.034	90
RE	.925	.880	.037	98	.929	.900	.035	93
MN	.941	.908	.038	73	.953	.921	.036	68

TABLE II.4 (Continued)

Case 4a: k=5, p_i=0.977, n_i=15, i=1,2,...,5. R_s=0.890

FIRST METHOD: Fbar = 1.8, Nfail = 84

	*******90% CL**********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.907	.786	.096	83	.879	.819	.087	53
MD	.872	.760	.089	83	.866	.808	.084	83
LG	.708	.614	.057	83	.780	.721	.066	83
MML	.778	.671	.076	83	.765	.720	.074	83
RE	.818	.706	.083	100	.825	.754	.080	97
MN	.810	.725	.069	83	.813	.771	.069	83

Case 4b: Same as 4a

SECOND METHOD: Fbar = 1.9, Ncorr = 1754

ML	.939	.780	.103	83	.861	.813	.093	65
MD	.901	.754	.095	83	.849	.802	.090	53
LG	.652	.609	.044	81	.759	.717	.064	83
MML	.778	.665	.074	83	.751	.716	.072	83
RE	.818	.701	.081	100	.815	.750	.079	97
MN	.885	.754	.089	83	.837	.791	.084	83

TABLE II.5 (Continued)

Case 5a: Same as 4a, except $n_i=30$, $i=1,2,\ldots,5$

FIRST METHOD : Fbar = 3.5, Nfail = 10

	*******90% CL**********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.925	.818	.068	75	.897	.842	.063	70
MD	.907	.805	.065	87	.890	.836	.062	85
LG	.795	.741	.046	98	.848	.796	.052	95
MML	.842	.754	.058	98	.834	.787	.056	98
RE	.853	.773	.061	98	.856	.805	.058	92
MN	.877	.799	.055	98	.877	.826	.054	75

Case 5b : Same as 5a

SECOND METHOD: Fbar = 3.6, Ncorr = 1217

ML	.919	.816	.068	76	.893	.840	.063	70
MD	.902	.803	.065	87	.887	.834	.062	83
LG	.818	.744	.050	98	.845	.797	.055	89
MML	.837	.753	.057	98	.834	.787	.055	98
RE	.849	.772	.060	98	.856	.804	.057	93
MN	.902	.808	.062	87	.883	.832	.059	76

TABLE II.6 (Continued)

Case 6a: Same as 4a, except $n_i=50$, $i=1,2,\ldots,5$

FIRST METHOD: Fbar = 5.7, Nfail = 1

	********90% CL**********				**********80% CL ********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.899	.838	.052	82	.891	.856	.049	71
MD	.889	.830	.051	91	.887	.852	.048	81
LG	.851	.796	.045	98	.864	.831	.045	89
MML	.855	.789	.047	98	.855	.823	.045	92
RE	.872	.809	.048	95	.874	.833	.045	89
MN	.890	.832	.047	87	.886	.850	.045	82

Case 6b: Same as 6a

SECOND METHOD: Fbar = 5.7, Ncorr = 779

ML	.898	.837	.052	82	.891	.855	.049	79
MD	.889	.829	.050	90	.887	.852	.048	79
LG	.851	.796	.045	98	.864	.830	.045	90
MML	.855	.798	.047	98	.855	.822	.045	92
RE	.872	.809	.047	95	.874	.833	.045	89
MN	.892	.835	.049	82	.886	.852	.047	81

TABLE II.7 (Continued)

Case 7a: p_i=0.961, n_i=15, i=1,2,...,5 R_s=0.820

FIRST METHOD: Fbar = 3.0, Nfail = 25

	*********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.851	.700	.115	79	.879	.740	.107	63
MD	.820	.680	.107	79	.866	.731	.104	78
LG	.631	.570	.067	95	.742	.662	.082	95
MML	.716	.605	.088	95	.765	.655	.087	95
RE	.766	.636	.094	97	.772	.686	.092	91
MN	.765	.660	.088	95	.813	.706	.089	79

Case 7b: Same as 7a

SECOND METHOD: Fbar = 3.1, Ncorr = 1381

ML	.831	.692	.115	79	.861	.732	.107	76
MD	.802	.673	.106	82	.849	.724	.103	79
LG	.652	.567	.065	91	.755	.657	.082	95
MML	.700	.601	.085	95	.751	.652	.084	95
RE	.753	.633	.091	97	.762	.683	.090	94
MN	.801	.677	.103	84	.837	.717	.099	79

TABLE II.8 (Continued)

Case 8a: Same as 7a, except n_i=30, for all i, i=1, 2,...,5

FIRST METHOD: Fbar = 5.6, Nfail = 2

	*********90%	CL**********			********80%	CL**********		
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.835	.742	.078	83	.823	.771	.074	69
MD	.822	.732	.075	84	.818	.766	.073	83
LG	.764	.683	.061	99	.784	.734	.065	89
MML	.772	.689	.067	91	.771	.724	.066	91
RE	.801	.706	.070	93	.794	.741	.068	86
MN	.821	.734	.070	87	.815	.762	.068	83

Case 8b: Same as 8a

SECOND METHOD: Fbar = 5.7, Ncorr = 768

ML	.833	.741	.078	83	.822	.769	.073	71
MD	.820	.731	.075	88	.817	.765	.072	81
LG	.762	.684	.064	99	.783	.734	.066	89
MML	.772	.689	.067	91	.771	.724	.065	91
RE	.800	.706	.069	94	.794	.740	.067	86
MN	.825	.738	.073	83	.816	.765	.070	82

TABLE II.9 (Continued)

Case 9a: Same as 7a except n_i=50 for all i, i=1,2,...,5.

FIRST METHOD: Fbar = 9.6 Nfail = 0

	*********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.829	.757	.060	84	.827	.779	.057	76
MD	.821	.751	.058	85	.824	.777	.057	76
LG	.790	.725	.054	95	.804	.759	.054	84
MML	.792	.726	.054	95	.796	.752	.053	87
RE	.808	.736	.055	92	.803	.762	.054	86
MN	.826	.757	.057	85	.824	.778	.055	76

Case 9b: Same as 9a

SECOND METHOD: Fbar = 9.6, Ncorr = 376

ML	.828	.757	.060	84	.827	.779	.057	76
MD	.820	.751	.058	87	.824	.777	.057	76
LG	.790	.724	.054	96	.804	.759	.054	84
MML	.792	.726	.054	95	.796	.752	.053	87
RE	.808	.736	.055	92	.803	.762	.054	86
MN	.827	.758	.057	84	.825	.779	.056	76

TABLE II.10 (Continued)

Case 10a: $k=5$, $p_i=0.950$, $n_i=15$, $i=1,2,...,5$. $R_s=0.774$

FIRST METHOD: Fbar = 3.7, Nfail = 14

	*******90% CL**********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.851	.650	.123	76	.799	.693	.116	72
MD	.820	.634	.114	88	.789	.680	.112	72
LG	.631	.542	.077	97	.701	.626	.091	95
MML	.716	.567	.094	97	.699	.618	.094	89
RE	.723	.597	.099	95	.727	.647	.098	90
MN	.765	.621	.099	89	.765	.667	.100	76

Case 10b: Same as 10a

SECOND METHOD: Fbar = 3.8, Ncorr = 1145

ML	.831	.643	.121	86	.788	.687	.114	72
MD	.802	.628	.112	89	.778	.680	.111	76
LG	.619	.538	.076	96	.704	.621	.091	93
MML	.700	.565	.091	97	.699	.616	.092	89
RE	.716	.595	.097	96	.724	.645	.096	90
MN	.801	.633	.110	89	.770	.674	.107	79

TABLE II.11 (Continued)

Case 11a: Same as 10a, except n_i=30, i=1,2,...5.

FIRST METHOD: Fbar = 7.6, Nfail = 0

	********90% CL**********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.794	.681	.084	83	.787	.712	.081	76
MD	.782	.673	.081	88	.783	.708	.080	76
LG	.729	.633	.071	98	.750	.681	.074	88
MML	.737	.636	.073	96	.738	.672	.072	89
RE	.742	.652	.075	95	.745	.687	.074	89
MN	.776	.677	.078	88	.778	.707	.076	77

Case 11b: Same as 11a

SECOND METHOD: Fbar = 7.6, Ncorr = 551

ML	.791	.680	.084	86	.786	.711	.081	76
MD	.779	.672	.081	88	.781	.707	.079	76
LG	.726	.632	.071	98	.748	.680	.073	88
MML	.737	.636	.072	96	.738	.672	.072	89
RE	.742	.652	.075	95	.745	.687	.074	89
MN	.786	.680	.080	88	.781	.708	.078	76

TABLE II.12 (Continued)

Case 12a: Same as 10a, except n_i=50, i=1,2,...,5.

FIRST METHOD: Fbar = 12.5, Nfail = 0

	********90% CL*********				*********80% CL*********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.785	.703	.061	89	.770	.727	.059	81
MD	.778	.698	.060	89	.768	.725	.059	81
LG	.750	.676	.056	97	.750	.710	.056	89
MML	.750	.676	.056	95	.745	.704	.055	89
RE	.754	.686	.056	95	.760	.713	.056	86
MN	.782	.706	.058	89	.770	.728	.057	81

Case 12b: Same as 12a

SECOND METHOD: Fbar = 12.5, Ncorr = 187

ML	.784	.703	.061	89	.770	.727	.059	81
MD	.778	.689	.059	89	.768	.725	.058	81
LG	.749	.675	.056	97	.750	.709	.056	89
MML	.750	.676	.056	95	.745	.704	.055	89
RE	.754	.686	.056	95	.760	.713	.056	86
MN	.785	.706	.059	89	.770	.728	.058	81

TABLE II.13 (Continued)

Case 13a: k-5, p_i=0.947, N_i=15, i=1,...,5. R_s=0.762

FIRST METHOD: Fbar = 3.9, Nfail = 7

	********90% CL**********				*********80% CL*********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.762	.641	.120	77	.799	.684	.114	63
MD	.738	.626	.111	89	.789	.677	.110	74
LG	.627	.535	.075	98	.701	.619	.089	97
MML	.648	.559	.091	98	.699	.610	.092	91
RE	.714	.590	.097	96	.729	.640	.096	89
MN	.721	.614	.098	91	.765	.660	.098	77

Case 13b: Same as 13a

SECOND METHOD: Fbar = 4.0, Ncorr =1122

ML	.750	.634	.117	87	.788	.677	.111	74
MD	.727	.619	.108	89	.778	.671	.107	74
LG	.617	.533	.076	95	.704	.614	.089	93
MML	.648	.557	.089	98	.699	.608	.089	91
RE	.707	.588	.094	98	.726	.638	.094	92
MN	.731	.624	.106	79	.770	.665	.104	75

TABLE II.14 (Continued)

Case 14a: Same as 13a, except n_i=30, i=1,2,...,5.

FIRST METHOD: Fbar = 8.0, Nfail = 0

	********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.794	.668	.085	88	.785	.700	.081	78
MD	.782	.661	.081	88	.780	.697	.080	77
LG	.729	.623	.072	96	.748	.671	.075	88
MML	.737	.626	.073	96	.738	.661	.072	89
RE	.743	.641	.075	94	.743	.677	.074	86
MN	.776	.666	.078	88	.771	.696	.077	79

Case 14b: Same as 14a

SECOND METHOD: Fbar = 8.0, Ncorr = 493

ML	.791	.668	.084	88	.782	.699	.081	78
MD	.779	.660	.081	88	.778	.696	.080	77
LG	.726	.622	.072	98	.745	.670	.074	88
MML	.737	.626	.073	96	.738	.661	.072	89
RE	.743	.641	.075	94	.743	.677	.074	86
MN	.786	.668	.080	88	.778	.697	.078	78

TABLE II.15 (Continued)

Case 15a: Same as 13a, except n_i=50, i=1,2,...,5.

FIRST METHOD: Fbar = 13.2, Nfail = 0

	*******90% CL**********				*********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.766	.690	.060	85	.768	.715	.059	77
MD	.760	.685	.059	92	.766	.713	.058	77
LG	.733	.664	.055	97	.749	.697	.056	85
MML	.734	.665	.055	97	.741	.692	.054	86
RE	.743	.674	.056	95	.745	.701	.055	85
MN	.767	.693	.058	86	.768	.715	.057	77

Case 15b: Same as 15a

SECOND METHOD: Fbar =13.2, Ncorr = 169

ML	.766	.690	.060	85	.768	.715	.059	77
MD	.760	.685	.059	92	.766	.713	.058	77
LG	.733	.663	.055	97	.748	.697	.056	85
MML	.734	.665	.055	97	.741	.692	.054	86
RE	.743	.674	.056	95	.745	.701	.055	85
MN	.767	.694	.058	85	.768	.716	.057	77

TABLE II.16 (Continued)

Case 16a: $k=15$, $p_i=0.995$, $n_i=5$, $i=1,2,...,5$. $R_s=0.928$

FIRST METHOD: Fbar = 1.3, Nfail = 157

	********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.907	.831	.078	68	.928	.860	.071	68
MD	.872	.801	.072	68	.912	.847	.068	68
LG	.708	.632	.057	68	.818	.748	.058	68
MML	.778	.707	.064	68	.824	.756	.063	68
RE	.839	.745	.072	100	.850	.792	.069	100
MN	.810	.757	.053	68	.855	.803	.053	68

Case 16b: Same as 16a

SECOND METHOD: Fbar = 1.4, Ncorr = 6952

ML	.939	.827	.094	68	.954	.856	.084	68
MD	.901	.798	.087	68	.938	.843	.081	68
LG	.646	.620	.030	68	.797	.744	.053	68
MML	.778	.701	.064	68	.824	.750	.063	68
RE	.839	.740	.072	100	.850	.787	.069	100
MN	.895	.804	.079	68	.920	.836	.074	68

TABLE II.17 (Continued)

Case 17a: Same as 16a, except $n_i=30$, $i=1,2,...,15$

FIRST METHOD: Fbar = 2.3, Nfail = 37

	********90% CL***********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.925	.867	.058	68	.939	.887	.053	68
MD	.907	.852	.055	92	.932	.880	.052	68
LG	.796	.764	.035	92	.861	.830	.042	92
MML	.842	.796	.049	92	.871	.828	.047	92
RE	.880	.816	.052	100	.889	.846	.050	97
MN	.877	.840	.043	92	.904	.865	.042	92

Case 17b: Same as 17a

SECOND METHOD: Fbar = 2.4, Ncorr = 6426

ML	.918	.865	.061	76	.933	.884	.055	68
MD	.901	.850	.058	81	.926	.878	.054	87
LG	.816	.777	.039	92	.877	.835	.046	92
MML	.837	.795	.048	92	.867	.826	.046	92
RE	.876	.815	.051	100	.886	.845	.049	97
MN	.906	.857	.054	76	.923	.878	.051	76

TABLE II.18 (Continued)

Case 18a: Same as 16a, except n_i=50, i=1,2,...,50

FIRST METHOD: Fbar = 3.8, Nfail = 18

	********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.955	.883	.046	74	.937	.898	.043	73
MD	.944	.874	.045	88	.933	.894	.042	73
LG	.893	.832	.037	96	.899	.868	.039	88
MML	.901	.840	.042	96	.896	.862	.039	88
RE	.904	.851	.043	96	.909	.873	.040	91
MN	.925	.873	.038	88	.925	.890	.037	74

Case 18b: Same as 18a

SECOND METHOD: Fbar = 3.8, Ncorr = 5851

ML	.952	.882	.048	79	.936	.897	.044	73
MD	.941	.873	.046	88	.931	.893	.043	73
LG	.888	.834	.038	96	.904	.869	.040	95
MML	.899	.839	.041	96	.896	.862	.039	88
RE	.902	.851	.043	96	.909	.873	.040	91
MN	.944	.880	.043	83	.931	.895	.041	73

TABLE II.19 (Continued)

Case 19a: k=10, p_i=0.990, n_i=15, i=12,...,10. R_s=0.904

FIRST METHOD: Fbar = 1.6, Nfail = 110

	*******90% CL**********				********80% CL***********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.907	.801	.090	78	.928	.833	.082	78
MD	.872	.774	.083	78	.912	.821	.079	78
LG	.708	.618	.056	78	.818	.729	.063	78
MML	.778	.684	.072	78	.824	.733	.070	78
RE	.820	.719	.079	100	.833	.766	.076	100
MN	.810	.736	.062	78	.855	.782	.062	78

Case 19b: Same as 19a

SECOND METHOD: Fbar = 1.8, Ncorr = 4289

ML	.939	.794	.102	78	.954	.826	.091	72
MD	.901	.768	.094	78	.938	.815	.088	78
LG	.648	.611	.037	88	.797	.725	.061	78
MML	.778	.678	.071	78	.824	.728	.069	78
RE	.820	.714	.078	100	.833	.762	.075	100
MN	.892	.773	.087	78	.919	.808	.082	78

TABLE II.20 (Continued)

Case 20a: Same as 19a, except N_i=30, i=1,2,...,10

FIRST METHOD: Fbar = 3.0, Nfail = 23

	*******90% CL*********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.925	.839	.066	81	.897	.861	.061	59
MD	.907	.825	.064	81	.890	.855	.060	80
LG	.796	.751	.043	95	.849	.811	.050	95
MML	.842	.773	.057	95	.834	.805	.054	95
RE	.869	.791	.059	98	.868	.822	.056	93
MN	.877	.818	.053	95	.877	.844	.052	81

Case 20b: Same as 20a

SECOND METHOD: Fbar = 3.1, Ncorr = 3703

ML	.919	.837	.068	81	.892	.859	.062	78
MD	.901	.823	.065	83	.886	.853	.061	81
LG	.816	.758	.048	95	.844	.813	.054	95
MML	.837	.772	.056	95	.834	.804	.054	95
RE	.865	.790	.058	98	.868	.821	.055	94
MN	.905	.831	.062	81	.885	.853	.058	81

TABLE II.21 (Continued)

Case 21a: Same as 19a, except $n_i=50$, $i=1,2,...,10$

FIRST METHOD: Fbar = 5.0, Nfail = 5

	********90% CL**********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.925	.853	.048	80	.913	.870	.044	76
MD	.915	.844	.046	89	.909	.867	.044	76
LG	.856	.809	.039	99	.880	.843	.040	94
MML	.876	.812	.043	96	.874	.836	.041	89
RE	.879	.823	.044	97	.881	.846	.042	91
MN	.910	.847	.042	89	.902	.865	.040	80

Case 21b: Same as 21a

SECOND METHOD: Fbar = 5.1, Ncorr = 3019

ML	.923	.852	.048	86	.911	.869	.045	76
MD	.913	.843	.047	89	.907	.866	.044	76
LG	.869	.809	.040	99	.883	.843	.041	89
MML	.876	.812	.043	96	.874	.836	.041	89
RE	.879	.823	.044	97	.881	.846	.042	91
MN	.918	.851	.045	86	.908	.868	.042	76

TABLE II.22 (Continued)

Case 22a: k=15, p_i=0.990, i=1,2,...,14, p_{15}=0.995; n_i=15, i=1,2,...,15. R_s=0.864

FIRST METHOD: Fbar = 2.3, Nfail = 62

	*********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.907	.755	.110	62	.879	.791	.102	62
MD	.872	.732	.102	87	.866	.781	.098	62
LG	.708	.589	.061	87	.742	.695	.076	87
MML	.778	.648	.085	87	.765	.698	.084	87
RE	.796	.683	.093	99	.815	.731	.090	94
MN	.810	.704	.080	87	.813	.749	.080	87

Case 22b: Same as 22a

SECOND METHOD: Fbar = 2.4, Ncorr = 6499

ML	.939	.744	.116	76	.859	.781	.106	76
MD	.901	.722	.107	87	.846	.771	.103	78
LG	.646	.590	.055	90	.755	.692	.077	87
MML	.778	.643	.083	87	.751	.693	.082	87
RE	.796	.678	.091	99	.805	.727	.089	94
MN	.895	.732	.102	87	.841	.768	.097	74

TABLE II.23 (Continued)

Case 23a: Same as 22a, except n_i=30, i=1,2,...,15

FIRST METHOD: Fbar =4.3, Nfail = 6

	********90% CL**********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.877	.793	.072	78	.897	.818	.067	66
MD	.862	.781	.069	92	.890	.813	.066	78
LG	.772	.719	.050	99	.837	.774	.056	92
MML	.803	.733	.061	99	.834	.767	.059	92
RE	.833	.751	.064	95	.839	.784	.062	90
MN	.853	.780	.060	79	.877	.807	.059	78

Case 23b: Same as 23a

SECOND METHOD: Fbar = 4.3, Ncorr = 5644

ML	.872	.789	.072	78	.892	.815	.067	76
MD	.857	.777	.069	92	.886	.810	.066	76
LG	.789	.723	.056	99	.842	.775	.059	92
MML	.803	.733	.061	99	.834	.767	.059	92
RE	.833	.751	.064	96	.839	.784	.062	90
MN	.865	.789	.067	79	.886	.812	.064	76

TABLE II.24 (Continued)

Case 24a: Same as 22a, except n_i=50, i=1,2,...,15

FIRST METHOD: Fbar = 7.3, Nfail = 0

	*********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.875	.805	.053	85	.869	.825	.050	75
MD	.866	.797	.052	85	.865	.822	.050	76
LG	.830	.767	.046	99	.843	.801	.047	90
MML	.832	.769	.048	94	.834	.794	.046	85
RE	.848	.780	.049	94	.843	.805	.047	88
MN	.869	.804	.049	85	.865	.823	.047	77

Case 24b: Same as 24a

SECOND METHOD: Fbar = 7.3, Ncorr = 4628

ML	.873	.804	.053	85	.868	.824	.050	75
MD	.864	.796	.052	92	.864	.821	.050	81
LG	.829	.767	.047	98	.842	.801	.047	92
MML	.832	.769	.048	98	.834	.794	.046	93
RE	.848	.780	.049	94	.843	.804	.047	88
MN	.873	.807	.050	85	.867	.825	.048	75

TABLE II.25 (Continued)

Case 25a: $k=15$, $p_i=0.990$, $N_i=15$, $i=1,2,...,15$. $R_s=0.860$

FIRST METHOD: Fbar = 2.3, Nfail = 46

	********90% CL**********				*********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.851	.751	.103	65	.879	.787	.095	65
MD	.820	.728	.095	91	.866	.777	.091	65
LG	.681	.587	.054	91	.742	.692	.069	91
MML	.716	.645	.078	91	.765	.695	.077	91
RE	.783	.678	.084	99	.804	.727	.082	95
MN	.765	.702	.073	91	.813	.747	.074	91

Case 25b: Same as 25a

SECOND METHOD: Fbar = 2.5, Ncorr = 6460

ML	.828	.739	.107	81	.859	.776	.098	81
MD	.799	.716	.099	91	.846	.766	.095	80
LG	.646	.590	.052	92	.755	.689	.071	91
MML	.700	.640	.076	91	.751	.690	.075	91
RE	.773	.674	.082	99	.793	.723	.080	95
MN	.808	.728	.094	91	.841	.764	.090	75

TABLE II.26 (Continued)

Case 26a: Same as 25a, except $n_i=30$, i=1,2,...,15

FIRST METHOD: Fbar = 4.5, Nfail = 6

	********90% CL*********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.877	.785	.077	81	.858	.810	.072	67
MD	.862	.773	.074	82	.853	.805	.070	80
LG	.772	.713	.054	99	.815	.767	.061	93
MML	.803	.726	.065	93	.805	.760	.063	93
RE	.832	.744	.067	96	.835	.777	.065	89
MN	.853	.773	.065	82	.846	.800	.064	81

Case 26b: Same as 25a

SECOND METHOD: Fbar = 4.6, Ncorr = 5591

ML	.872	.781	.077	81	.855	.807	.072	78
MD	.857	.770	.073	87	.849	.802	.070	79
LG	.789	.716	.060	99	.811	.768	.063	93
MML	.803	.726	.065	99	.805	.760	.063	93
RE	.832	.744	.067	96	.835	.777	.065	89
MN	.865	.781	.071	81	.851	.805	.068	78

TABLE II.27 (Continued)

Case 27a: Same as 25a, except n_i=50, i-1,2,...,15

FIRST METHOD: Fbar = 7.4, Nfail = 1

	*******90% CL*********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.875	.802	.057	80	.869	.822	.054	74
MD	.866	.795	.055	86	.865	.819	.053	74
LG	.823	.765	.049	98	.844	.799	.050	86
MML	.832	.767	.051	94	.834	.792	.050	86
RE	.841	.778	.052	95	.846	.802	.050	89
MN	.869	.802	.052	86	.865	.821	.051	74

Case 27b: Same as 27a

SECOND METHOD: Fbar = 7.5, Ncorr = 4577

ML	.873	.801	.056	86	.868	.821	.054	74
MD	.864	.794	.055	86	.864	.818	.053	74
LG	.829	.765	.050	98	.842	.799	.050	86
MML	.832	.767	.051	94	.834	.792	.050	86
RE	.841	.777	.052	95	.846	.802	.050	89
MN	.873	.805	.054	86	.867	.822	.052	74

TABLE II.28 (Continued)

Case 28a: k=15, p_i=0.995, i=1,2,...,14, p_{15}=0.850, n_i=15, i=1,2,...,15. R_s=0.792

FIRST METHOD: Fbar = 3.2, Nfail = 16

	*******90% CL*********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.851	.678	.124	72	.799	.719	.116	63
MD	.820	.659	.115	82	.789	.711	.112	82
LG	.681	.563	.081	97	.719	.648	.093	87
MML	.716	.585	.098	97	.699	.636	.097	82
RE	.753	.618	.103	97	.757	.668	.102	88
MN	.765	.635	.100	97	.765	.683	.100	82

Case 28b: Same as 28a

SECOND METHOD: Fbar = 3.4, Ncorr = 6548

ML	.828	.664	.123	82	.783	.707	.116	76
MD	.799	.647	.114	82	.774	.699	.112	78
LG	.641	.548	.072	98	.698	.635	.088	97
MML	.700	.578	.096	97	.699	.629	.096	97
RE	.739	.611	.102	97	.750	.662	.101	90
MN	.808	.663	.110	82	.773	.701	.107	76

TABLE II.29 (Continued)

Case 29a: Same as 28a, except n_i=30, i=1,2,...,15

FIRST METHOD: Fbar = 6.6, Nfail = 1

	*******90% CL**********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.796	.702	.081	83	.790	.733	.078	80
MD	.783	.693	.078	91	.785	.729	.076	80
LG	.731	.651	.067	98	.754	.701	.069	91
MML	.737	.653	.071	98	.745	.689	.070	92
RE	.761	.670	.073	96	.768	.705	.071	89
MN	.784	.694	.073	92	.785	.724	.072	80

Case 29b: Same as 29a

SECOND METHOD: Fbar = 6.7, Ncorr = 5985

ML	.792	.699	.081	90	.787	.730	.077	80
MD	.780	.690	.078	91	.783	.726	.076	80
LG	.726	.648	.066	99	.750	.698	.069	92
MML	.737	.652	.071	98	.745	.688	.070	92
RE	.761	.669	.072	96	.766	.705	.071	89
MN	.792	.706	.076	89	.786	.732	.074	80

TABLE II.30 (Continued)

Case 30a: Same as 28a, except $n_i=50$, $i=1,2,\ldots,15$

FIRST METHOD: Fbar = 10.9, Nfail = 0

	********90% CL**********				*********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.803	.723	.061	87	.800	.747	.059	78
MD	.796	.718	.059	88	.797	.745	.058	78
LG	.766	.695	.054	98	.777	.729	.054	87
MML	.766	.693	.056	97	.768	.721	.055	89
RE	.774	.703	.057	95	.775	.731	.056	87
MN	.800	.724	.057	88	.793	.746	.056	79

Case 30b: Same as 30a

SECOND METHOD: Fbar = 11.0, Ncorr = 5499

ML	.802	.722	.061	87	.798	.746	.058	78
MD	.795	.717	.059	89	.796	.744	.058	78
LG	.764	.694	.054	98	.778	.728	.054	87
MML	.766	.693	.056	97	.768	.720	.055	89
RE	.773	.703	.057	95	.775	.730	.056	87
MN	.807	.731	.058	87	.801	.751	.056	78

TABLE II.31 (Continued)

Case 31a: k=13, R_s=0.723

p_i: .995 .985 .979 .988 .982 .980 .967 .995 .970
.995 .968 .980 .900

n_i=15, i=1,2,...,13.

FIRST METHOD: Fbar = 4.5, Nfail = 5

	*******90% CL*********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.762	.602	.123	84	.731	.647	.118	68
MD	.738	.589	.114	84	.723	.641	.114	84
LG	.598	.510	.081	99	.663	.588	.095	89
MML	.648	.531	.094	93	.655	.581	.095	84
RE	.685	.560	.098	94	.690	.610	.098	88
MN	.721	.583	.102	86	.709	.628	.103	84

Case 31b: Same as 31a

SECOND METHOD: Fbar = 4.7, Ncorr = 4779

ML	.745	.591	.120	84	.718	.637	.115	77
MD	.723	.579	.112	91	.711	.631	.111	74
LG	.607	.502	.080	99	.648	.579	.093	93
MML	.648	.526	.094	93	.655	.577	.095	86
RE	.685	.555	.098	95	.689	.605	.099	88
MN	.735	.593	.110	84	.711	.633	.108	77

TABLE II.32 (Continued)

Case 32a: Same as 31a, except n_i=30, i=1,2,...,13

FIRST METHOD: Fbar = 9.3, Nfail = 0

	*******90% CL*********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.728	.631	.080	85	.727	.663	.078	75
MD	.718	.625	.077	90	.723	.661	.077	82
LG	.675	.590	.069	96	.695	.637	.072	88
MML	.678	.594	.069	96	.682	.630	.069	86
RE	.701	.609	.071	93	.704	.644	.071	86
MN	.725	.633	.075	87	.724	.663	.074	79

Case 32b : Same as 32a

SECOND METHOD: Fbar = 9.4, Ncorr = 3673

ML	.725	.629	.080	87	.724	.661	.077	78
MD	.716	.623	.077	90	.721	.659	.076	84
LG	.672	.588	.069	97	.692	.635	.071	90
MML	.678	.593	.069	96	.682	.629	.069	86
RE	.701	.608	.071	93	.704	.644	.071	86
MN	.729	.637	.076	85	.725	.665	.075	76

TABLE II.33 (Continued)

Case 33a : Same as 31a, except $n_i=50$, $i=1,2,...,13$

FIRST METHOD: Fbar = 15.6, Nfail = 0

	*********90% CL********				********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.729	.652	.063	84	.733	.678	.061	77
MD	.724	.648	.061	87	.731	.676	.061	77
LG	.700	.628	.058	95	.714	.662	.058	84
MML	.699	.630	.057	93	.707	.658	.057	84
RE	.710	.639	.058	93	.716	.667	.057	82
MN	.733	.659	.060	84	.734	.681	.059	77

Case 33b: Same as 33a

SECOND METHOD: Fbar = 15.6, Ncorr = 2752

ML	.729	.652	.063	84	.732	.677	.061	77
MD	.723	.648	.061	90	.730	.676	.061	77
LG	.699	.628	.058	95	.713	.661	.058	84
MML	.699	.630	.057	93	.707	.658	.057	84
RE	.709	.639	.058	93	.716	.666	.057	82
MN	.735	.660	.061	84	.736	.682	.060	77

TABLE II.34 (Continued)

Case 34a: k=13. R_s=0.723

p_i : .995 .985 .979 .988 .982 .980 .967 .995
.970 .995 .968 .980 .900

n_i: 150 90 75 100 125 18 28 125
63 125 59 5 19

FIRST METHOD: Fbar = 15.5, Nfail = 0

	*******90% CL **********				*******80% CL***********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.749	.621	.110	81	.740	.656	.101	72
MD	-	-	-	-	-	-	-	-
LG	.721	.594	.108	91	.724	.641	.099	80
MML	.721	.594	.109	90	.716	.631	.102	81
RE	.737	.611	.104	86	.733	.647	.097	77
MN	.491	.404	.065	100	.549	.484	.070	100

Case 34b: Same as 34a

SECOND METHOD: Fbar = 15.6, Ncorr = 2425

ML	.723	.603	.102	90	.720	.640	.095	80
MD	.656	.559	.086	99	.699	.624	.089	88
LG	.683	.570	.097	97	.700	.624	.091	87
MML	.685	.571	.098	97	.687	.611	.094	91
RE	.703	.590	.095	94	.708	.630	.090	85
MN	.553	.453	.078	100	.594	.522	.080	100

TABLE II.35 (Continued)

Case 35a: k=15. R_s=0.860

p_i: .990 .990 .990 .990 .990 .990 .990 .990

.990 .990 .990 .990 .990 .990 .990

n_i: 250 40 120 15 130 65 70 75

100 90 60 60 20 30 40

FIRST METHOD: Fbar = 11.6, Nfail = 0

	*******90% CL*********				*******80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.881	.803	.064	80	.874	.823	.059	72
MD	-	-	-	-	-	-	-	-
LG	.861	.776	.067	89	.862	.809	.060	79
MML	.862	.778	.068	89	.856	.802	.062	82
RE	.870	.788	.066	86	.863	.812	.060	78
MN	.780	.720	.049	100	.803	.762	.048	99

Case 35b: Same as 35a

SECOND METHOD: Fbar = 11.6, Ncorr = 3881

ML	.876	.800	.063	82	.870	.820	.058	73
MD	.855	.784	.060	91	.865	.815	.057	76
LG	.854	.772	.065	92	.857	.806	.058	81
MML	.856	.774	.066	91	.853	.799	.061	83
RE	.864	.785	.064	88	.859	.809	.059	80
MN	.811	.743	.056	99	.824	.778	.053	95

TABLE II.36 (Concluded)

Case 36a: k=15. R_s=0.792

p_i: .995 .995 .995 .995 .995 .995 .995 .995

.995 .995 .995 .995 .995 .995 .850

n_i: 20 20 20 20 20 20 20 20

20 20 20 20 20 20 150

FIRST METHOD: Fbar = 23.9, Nfail = 0

	*******90% CL*********				*********80% CL**********			
	$A_{.90}$	m	s	ACL	$A_{.80}$	m	s	ACL
ML	.824	.732	.072	78	.808	.754	.066	71
MD	-	-	-	-	-	-	-	-
LG	.814	.714	.075	81	.803	.744	.067	75
MML	.812	.713	.075	81	.798	.737	.068	77
RE	.816	.721	.072	81	.804	.745	.066	75
MN	.733	.677	.048	99	.751	.713	.049	96

Case 36b: Same as 36a

SECOND METHOD: Fbar = 24.3, Ncorr = 6296

ML	.802	.711	.069	86	.791	.735	.064	80
MD	.788	.697	.067	91	.785	.730	.063	81
LG	.790	.692	.071	88	.781	.724	.064	84
MML	.789	.691	.071	88	.776	.718	.066	84
RE	.794	.701	.069	88	.785	.728	.064	83
MN	.749	.676	.058	97	.759	.709	.057	93

APPENDIX

THE COMPUTER PROGRAM

The computer program consists of two main programs and five subroutines. The two main programs are basically the same. The first main program is used with the FIRST METHOD of introducing partial component failures and the second main program is used with the SECOND METHOD of introducing partial component failures. The main program reads the input (case) k, n_i, p_i, $i=1,2,\ldots,k$. With this input the main program constructs binomial data and sends this data to subroutines MADSKY (Madansky), RMMLI (Easterling and Randomized Easterling), AMLMAN (Max. l'hood and Mann) and WOODBG (Log-Gamma); these subroutines compute lower confidence limits. For each case these computations are repeated 500 times. Then subroutine CMPARE computes the 90-th and 80-th percentile, mean, std. dev. and actual confidence level.

The programs are self-explanatory since comments are inserted throughout the programs. Note that subroutine RANDOM and PXSORT are from Ref. 10, subroutine MDBETI from Ref. 3. The percentile points of the chi-square and standard normal distributions are from Ref. 8.

The successful root finding technique in the MD procedure which was developed by Lisowsky (Ref. 4) is implemented in subroutine MADSKY.

```
C*****************FIRST MAIN PROGRAM************************************
C
C
C
C       K = SCALAR,NUMBER OF COMPONENTS IN SERIES SYSTEM RELIABILITY
C       N(I) = SAMPLE SIZE OF I-TH COMPONENT,
C       P(I) = THE TRUE COMPONENT'S RELIABILITY,
C       X(I) = NUMBER OF SUCCESSES,
C       F(I) = NUMBER OF FAILURES,
C       I = 1,2,....,K
C       RHOLR,RHOBIN,RHOAML,RHOMAN,RHOGAM AND RHOMCD ARE VECTORS OF TWO
C       COMPONENTS,CONTAIN THE LOWER CONFIDENCE LIMIT (LCL) AT 90% AND
C       80% LEVEL USING MADANSKY,EASTERLING,MAX L'HOOD,MANN,LOGGAMMA
C       AND RANDOMIZED EASTERLING PROCEDURE RESPECTIVELY
C
C
C
C       RHOMAD,RHOEAS,RHOMLE,RHOMCY,RHOWB AND RHOMML ARE TWO DIMENSIONAL
C       ARRAYS OF 2X500 SIZE,CONTAIN THE INFORMATION AS ABOVE WITH 500
C       REPLICATIONS
C
C
C
C       STDMAD,STDEAS,STDMLE,STDMCY,STDWB AND STDMML ARE VECTORS OF TWO
C       COMPONENTS,CONTAIN STD DEV. OF THE LCL DISTRIBUTION OF ALL PRO
C       CEDURES MENTIONED ABOVE
C
C
C
C       CTLMAD,CTLEAS,CTLMLE,CTLMCY,CTLWB AND CTLMML ARE VECTORS OF TWO
C       COMPONENTS,CONTAIN THE 90-TH AND 80-TH PERCENTILE OF ALL THE
C       LCL'S DISTRIBUTION
C
C
C
C       XBRMAD,XBREAS,XBRMLE,XBRMCY,XBRWB AND XBRMML ARE VECTORS OF TWO
C       COMPONENTS,CONTAIN THE MEAN OF ALL THE LCL'S DISTRIBUTION
C
C
C
C       INDMAD,INDEAS,INDMLE,INDMCY,INDWB AND INDMML ARE VECTORS OF TWO
C       COMPONENTS,CONTAIN THE PERCENTILE POINTS CLOSEST TO THE TRUE
C       SYSTEM RELIABILITY
C
C
      DIMENSION N(100),X(100),F(100),P(100)
      DIMENSION RHOBIN(2),RHOEAS(2,500),STDEAS(2),CTLEAS(2),XBREAS(2),
     *         INDEAS(2)
      DIMENSION RHOLR(2),RHOMAD(2,500),STDMAD(2),CTLMAD(2),XBRMAD(2),
     *         INDMAD(2)
```

```
      DIMENSION RHOAML(2),RHOMLE(2,500),STDMLE(2),CTLMLE(2),XBRMLE(2),
     *          INDMLE(2)
      DIMENSION RHOMAN(2),RHOMCY(2,500),STDMCY(2),CTLMCY(2),XBRMCY(2),
     *          INDMCY(2)
      DIMENSION RHOGAM(2),RHOWB(2,500),STDWB(2),CTLWB(2),XBRWB(2),
     *          INDWB(2)
      DIMENSION RHOMCC(2),RHOMML(2,500),STDMML(2),CTLMML(2),XBRMML(2),
     *          INDMML(2)
C
C
C
C         IS = SEED FOR SUBROUTINE RANDOM,IS USED TO GENERATE UNIFORMLY
C         DISTRIBUTED RANDOM NUMBER U(0,1),THIS RANDOM NUMBER IS USED TO
C         CONSTRUCT BINOMIAL DATA
C
C         ISEED = SEED AS ABOVE, THE RESULTING RANDOM NUMBER Y(0,1) IS SENT
C         TO SUBROUTINE RMMLI TO BE USED IN RANDOMIZED EASTERLING
C         PROCEDURE
C
C
C
C
      DATA IS,ISEED/260543,270377/
      CALL OVFLOW
C
      WRITE(6,900)
      WRITE(6,901)
      DO 999 ICASE = 1,100
         READ(5,920) K
            IF (K.EQ.99) GO TO 1000
         READ(5,921) (P(I),I=1,K)
         READ(5,922) (N(I),I=1,K)
C
C
         MINN = N(1)
         DO 55 J = 2,K
              IF (MINN.NE.N(J)) GO TO 66
   55    CONTINUE
C
C        IFLAG = 1 UNEQUAL SAMPLE SIZE
C        IFLAG = 0 EQUAL SAMPLE SIZE
C
         IFLAG = 0
         GO TO 77
   66    IFLAG = 1
   77    CONTINUE
C
C
C        RS = THE TRUE INDEPENDENT SERIES SYSTEM RELIABILITY
C
         RS=1.0
      NMAX = 1
      DO 100 I=1,K
         RS=RS*P(I)
C     MINN = SMALLEST SAMPLE SIZE
         IF (N(I).LT.MINN) MINN = N(I)
         CONTINUE
         IF (NMAX.GT.N(I)) GO TO 100
C     NMAX = LARGEST SAMPLE SIZE
         NMAX = N(I)
C     MAX = INDEX OF THE LARGEST SAMPLE SIZE
         MAX = I
  100 CONTINUE
C
C     NOFAIL = TOTAL NUMBER OF INTRODUCING PARTIAL FAILURE
C              USING THE FIRST METHOD
C
      NOFAIL=0
C
C     AVFAIL = AVERAGE FAILURE IN 500 REPLICATIONS
C
      AVFAIL = 0.0
      DO 120 IR=1,500
         CALL RANDOM(ISEED,Y,1)
         TOTN = 0.0
         SUMMX = 0.0
         DO 110 I=1,K
            M=N(I)
            X(I)=0.0
            DO 105 IM=1,M
               CALL RANDOM(IS,U,1)
               IF(U.LT.P(I)) X(I)=X(I)+1.0
  105       CONTINUE
         TOTN = TOTN + FLOAT(N(I))
         SUMMX = SUMMX + X(I)
  110    CONTINUE
         IF (TOTN.NE.SUMMX) GO TO 101
C
C
C
C     FIRST METHOD OF INTRODUCING PARTIAL COMPONENT FAILURE
C     SET X(MAX) = X(MAX) - 0.5, IF X(I) = N(I) FOR ALL I
C
C
         X(MAX) = X(MAX) - 0.5
         NOFAIL=NOFAIL+1
  101    CONTINUE
C
C     PSI = MAX. L'HOOD ESTIMATE OF SYSTEM RELIABILITY
C
         PSI = 1.0
```

```
C        AMINX = SMALLEST NUMBER OF SUCCESS
C
             AMINX = X(1)
             DO 111 J = 1,K
                  F(J) = FLOAT(N(J))-X(J)
                  AVFAIL = AVFAIL + F(J)
                  PSI = PSI*(X(J)/FLOAT(N(J)))
                  IF (X(J).LT.AMINX) AMINX = X(J)
                  CONTINUE
  111        CONTINUE
C
C
C
C
C        SUBROUTINES FOR COMPUTING LCL :
C        MADSKY = MADANSKY PROCEDURE
C        RMMLI = EASTERLING AND RANDOMIZED EASTERLING PROCEDURES
C        AMLMAN = MAX. L'HOOD AND MANN PROCEDURES
C        WOODBG = LOG-GAMMA PROCEDURE
C
C
C
C
             CALL MADSKY(X,N,K,AMINX,RHOLR)
             CALL RMMLI(X,N,K,PSI,RHOBIN,RHOMOD,Y,SIGMA2)
             CALL AMLMAN(F,X,N,K,MINN,PSI,RHOAML,RHOMAN,IFLAG,SIGMA2)
             CALL WOODBG(N,F,RHOGAM,K,MAX)
             DO 116 I=1,2
                  RHOMML(I,IR)=RHOMOD(I)
                  RHOEAS(I,IR)=RHOBIN(I)
                  RHOMAD(I,IR) = RHOLR(I)
                  RHOMLE(I,IR) = RHOAML(I)
                  RHOMCY(I,IR) = RHOMAN(I)
                  RHOWB(I,IR) = RHOGAM(I)
  116        CONTINUE
  120    CONTINUE
         AVFAIL = AVFAIL/500.0
C
C
C        CMPARE = SUBROUTINE FOR COMPUTING 90-TH AND 80-TH PERCENTILE
C                 POINTS,MEAN,STD. DEV.,AND PERCENTILE POINT CLOSEST TO
C                 THE TRUE SYSTEM RELIABILITY OF ALL THE ABOVE MENTIONED
C                 PROCEDURES
C
C
         CALL CMPARE(RHOMML,STDMML,CTLMML,INDMML,XBRMML,RS)
         CALL CMPARE(RHOEAS,STDEAS,CTLEAS,INDEAS,XBREAS,RS)
         CALL CMPARE(RHOMAD,STDMAD,CTLMAD,INDMAD,XBRMAD,RS)
         CALL CMPARE(RHOMLE,STDMLE,CTLMLE,INDMLE,XBRMLE,RS)
         CALL CMPARE(RHOMCY,STDMCY,CTLMCY,INDMCY,XBRMCY,RS)
         CALL CMPARE(RHOWB,STDWB,CTLWB,INDWB,XBRWB,RS)
         WRITE(6,902) ICASE,K
         WRITE(6,903) (P(I),I=1,K)
         WRITE(6,904) (N(I),I=1,K)
         WRITE(6,905) RS
         WRITE(6,906) AVFAIL
         WRITE(6,940) NOFAIL
         WRITE(6,907)
         WRITE(6,908) (CTLMLE(I),XBRMLE(I),STDMLE(I),INDMLE(I),I=1,2)
         WRITE(6,909) (CTLMAD(I),XBRMAD(I),STDMAD(I),INDMAD(I),I=1,2)
         WRITE(6,910) (CTLWB(I),XBRWB(I),STDWB(I),INDWB(I),I=1,2)
         WRITE(6,911) (CTLEAS(I),XBREAS(I),STDEAS(I),INDEAS(I),I=1,2)
         WRITE(6,930) (CTLMML(I),XBRMML(I),STDMML(I),INDMML(I),I=1,2)
         WRITE(6,912) (CTLMCY(I),XBRMCY(I),STDMCY(I),INDMCY(I),I=1,2)
         WRITE(6,913)
 999  CONTINUE
1000  STOP
 900  FORMAT('1')
 901  FORMAT(/////,39X,'A COMPARATIVE ANALYSIS OF SEVERAL METHODS FOR DE
     1TERMINING LOWER'///,40X,'CONFIDENCE LIMIT ON SERIES SYSTEM RELIABI
     2LITY USING COMPONENT'///,60X,'ATTRIBUTES TEST DATA')
 902  FORMAT(/////,6X,'CASE # ',24X,':',I3///,6X,'NUMBER OF COMPONENT',1
     12X,':',I3)
 903  FORMAT(///,6X,'COMPONENTS RELIABILITY',9X,':',1X,15(F4.3,1X))
 904  FORMAT(///,6X,'SAMPLE SIZE OF EACH COMPONENT  :',1X,15(I4,1X))
 905  FORMAT(///,6X,'SYSTEM RELIABILITY (RS)',8X,':',1X,F5.3)
 906  FORMAT(///,6X,'AVERAGE FAILURES PER REPLICA',3X,':',F5.1)
 907  FORMAT(/////,39X,'CONFIDENCE COEFFICIENT : 0.90',21X,'CONFIDENCE C
     1OEFFICIENT : 0.80'///,30X,'NINETIETH',4X,'MEAN',6X,'STD.',3X,'PERC
     2ENTILE POINT',4X,'EIGHTIETH',4X,'MEAN',6X,'STD.',3X,'PERCENTILE PO
     3INT'///,30X,'PERCENTILE',10X,'DEVIATION   CLOSEST TO RS',5X,'PERCE
     4NTILE',10X,'DEVIATION   CLOSEST TO RS')
 908  FORMAT(///,6X,'ASYMP. MAX. LIKELIHOOD  :',2(3(2X,F5.3,3X),5X,I3,12
     1X))
 909  FORMAT(///6X,'MADANSKY (1965)',9X,':',2(3(2X,F5.3,3X),5X,I3,12X))
 910  FORMAT(///,6X,'LOGGAMA (1968)',10X,':',2(3(2X,F5.3,3X),5X,I3,12X))
 911  FORMAT(///,6X,'EASTERLING (1972)',7X,':',2(3(2X,F5.3,3X),5X,I3,12X
     1))
 912  FORMAT(///,6X,'MANN (1974)',13X,':',2(3(2X,F5.3,3X),5X,I3,12X))
 913  FORMAT('1')
 920  FORMAT(I5)
 921  FORMAT(15F5.3)
 922  FORMAT(15I5)
 930  FORMAT(///,6X,'RANDOMIZED EASTERLING ',2X,':',2(3(2X,F5.3,3X),5X,I
     13,12X))
 940  FORMAT(///,6X,'TOTAL NUMBER OF NO FAILURES',4X,':',I5)
      END
```

```
C*****************SECOND MAIN PROGRAM**********************************
C
C
      DIMENSION N(100),X(100),R(100),P(100)
      DIMENSION RHOLR(2),RHOMAD(2,500),STDMAD(2),CTLMAD(2),XBRMAD(2),
     *          INDMAD(2)
      DIMENSION RHOAML(2),RHOMLE(2,500),STDMLE(2),CTLMLE(2),XBRMLE(2),
     *          INDMLE(2)
      DIMENSION RHOMAN(2),RHOMCY(2,500),STDMCY(2),CTLMCY(2),XBRMCY(2),
     *          INDMCY(2)
      DIMENSION RHOGAM(2),RHOWB(2,500),STDWB(2),CTLWB(2),XBRWB(2),
     *          INDWB(2)
      DIMENSION RHOBIN(2),RHOEAS(2,500),STDEAS(2),CTLEAS(2),XBREAS(2),
     *          INDEAS(2)
      DIMENSION RHOMOD(2),RHOMML(2,500),STDMML(2),CTLMML(2),XBRMML(2),
     *          INDMML(2)
      DATA IS,ISEED/260543,270377/
      CALL OVFLOW
      WRITE(6,900)
      WRITE(6,901)
      DO 999 ICASE = 1,100
         READ(5,920) K
           IF (K.EQ.99) GO TO 1000
         READ(5,921) (P(I),I=1,K)
         READ(5,922) (N(I),I=1,K)
         MINN = N(1)
         DO 55 J = 2,K
              IF (MINN.NE.N(J)) GO TO 66
   55    CONTINUE
         IFLAG = 0
         GO TO 77
   66    IFLAG = 1
   77    CONTINUE
         NMAX = 1
         RS=1.0
         DO 100 I=1,K
              RS=RS*P(I)
              IF (N(I).LT.MINN) MINN = N(I)
              CONTINUE
              IF (NMAX.GT.N(I)) GO TO 100
              NMAX = N(I)
              MAX = I
  100    CONTINUE
C        NOCORR = TOTAL NUMBER OF INTRODUCING PARTIAL FAILURE
C                 USING SECOND METHOD
         NOCORR=0
         AVFAIL = 0.0
         FK=FLOAT(K)
         DO 120 IR=1,500
              CALL RANDOM(ISEED,Y,1)
      DO 110 I=1,K
          M=N(I)
          X(I)=0.0
          TNI=FLOAT(N(I))
          DO 105 IM=1,M
              CALL RANDOM(IS,U,1)
              IF(U.LT.P(I)) X(I)=X(I)+1.0
  105     CONTINUE
      IF(TNI.NE.X(I)) GO TO 110
C     SECOND METHOD OF INTRODUCING PARTIAL COMPONENT FAILURE
C     SET X(I) = X(I)-4/(N(I)*K), WHENEVER X(I) = N(I)
      X(I)=X(I)-4.0/(TNI*FK)
      NOCORR=NOCORR+1
  110 CONTINUE
      PSI = 1.0
      AMINX = X(1)
      DO 111 J = 1,K
          F(J) = FLOAT(N(J))-X(J)
          AVFAIL = AVFAIL + F(J)
          PSI = PSI*(X(J)/FLOAT(N(J)))
          IF (X(J).LT.AMINX) AMINX = X(J)
          CONTINUE
  111 CONTINUE
      CALL MADSKY(X,N,K,AMINX,RHOLR)
      CALL RMMLI(X,N,K,PSI,RHOBIN,RHOMOD,Y,SIGMA2)
      CALL AMLMAN(F,X,N,K,MINN,PSI,RHOAML,RHOMAN,IFLAG,SIGMA2)
      CALL WOODBG(N,F,RHOGAM,K,MAX)
      DO 116 I=1,2
          RHOMML(I,IR)=RHOMOD(I)
          RHOEAS(I,IR)=RHOBIN(I)
          RHOMAD(I,IR) = RHOLR(I)
          RHOMLE(I,IR) = RHOAML(I)
          RHOMCY(I,IR) = RHOMAN(I)
          RHOWB(I,IR) = RHOGAM(I)
  116 CONTINUE
  120 CONTINUE
      AVFAIL = AVFAIL/500.0
      CALL CMPARE(RHOMML,STDMML,CTLMML,INDMML,XBRMML,RS)
      CALL CMPARE(RHOEAS,STDEAS,CTLEAS,INDEAS,XBREAS,RS)
      CALL CMPARE(RHOMAD,STDMAD,CTLMAD,INDMAD,XBRMAD,RS)
      CALL CMPARE(RHOMLE,STDMLE,CTLMLE,INDMLE,XBRMLE,RS)
      CALL CMPARE(RHOMCY,STDMCY,CTLMCY,INDMCY,XBRMCY,RS)
      CALL CMPARE(RHOWB,STDWB,CTLWB,INDWB,XBRWB,RS)
      WRITE(6,902) ICASE,K
      WRITE(6,903) (P(I),I=1,K)
      WRITE(6,904) (N(I),I=1,K)
      WRITE(6,905) RS
      WRITE(6,906) AVFAIL
      WRITE(6,950) NOCORR
      WRITE(6,907)
      WRITE(6,908) (CTLMLE(I),XBRMLE(I),STDMLE(I),INDMLE(I),I=1,2)
      WRITE(6,909) (CTLMAD(I),XBRMAD(I),STDMAD(I),INDMAD(I),I=1,2)
      WRITE(6,910) (CTLWB(I),XBRWB(I),STDWB(I),INDWB(I),I=1,2)
      WRITE(6,911) (CTLEAS(I),XBREAS(I),STDEAS(I),INDEAS(I),I=1,2)
      WRITE(6,930) (CTLMML(I),XBRMML(I),STDMML(I),INDMML(I),I=1,2)
      WRITE(6,912) (CTLMCY(I),XBRMCY(I),STDMCY(I),INDMCY(I),I=1,2)
      WRITE(6,913)
C
C
C
  999 CONTINUE
 1000 STOP
  900 FORMAT('1')
  901 FORMAT(/////,39X,'A COMPARATIVE ANALYSIS OF SEVERAL METHODS FOR DE
     1TERMINING LOWER'///,40X,'CONFIDENCE LIMIT ON SERIES SYSTEM RELIABI
     2LITY USING COMPONENT'///,60X,'ATTRIBUTES TEST DATA')
  902 FORMAT(/////,6X,'CASE # ',24X,':',I3///,6X,'NUMBER OF COMPONENT',1
     12X,':',I3)
  903 FORMAT(///,6X,'COMPONENTS RELIABILITY',9X,':',1X,15(F4.3,1X))
  904 FORMAT(///,6X,'SAMPLE SIZE OF EACH COMPONENT  :',1X,15(I4,1X))
  905 FORMAT(///,6X,'SYSTEM RELIABILITY (RS)',8X,':',1X,F5.3)
  906 FORMAT(///,6X,'AVERAGE FAILURES PER REPLICA',3X,':',F5.1)
  907 FORMAT(/////,39X,'CONFIDENCE COEFFICIENT : 0.90',21X,'CONFIDENCE C
     1OEFFICIENT : 0.80'///,30X,'NINETIETH',4X,'MEAN',6X,'STD.',3X,'PERC
     2ENTILE POINT',4X,'EIGHTIETH',4X,'MEAN',6X,'STD.',3X,'PERCENTILE PO
     3INT'///,30X,'PERCENTILE',10X,'DEVIATION   CLOSEST TO RS',5X,'PERCE
     4NTILE',10X,'DEVIATION   CLOSEST TO RS')
  908 FORMAT(///,6X,'ASYMP. MAX. LIKELIHOOD  :',2(3(2X,F5.3,3X),5X,I3,12
     1X))
  909 FORMAT(///6X,'MADANSKY (1965)',9X,':',2(3(2X,F5.3,3X),5X,I3,12X))
  910 FORMAT(///,6X,'LOGGAMA (1968)',10X,':',2(3(2X,F5.3,3X),5X,I3,12X))
  911 FORMAT(///,6X,'EASTERLING (1972)',7X,':',2(3(2X,F5.3,3X),5X,I3,12X
     1))
  912 FORMAT(///,6X,'MANN (1974)',13X,':',2(3(2X,F5.3,3X),5X,I3,12X))
  913 FORMAT('1')
  920 FORMAT(I5)
  921 FORMAT(15F5.3)
  922 FORMAT(15I5)
  930 FORMAT(///,6X,'RANDOMIZED EASTERLING ',2X,':',2(3(2X,F5.3,3X),5X,I
     13,12X))
  950 FORMAT(///,6X,'TOTAL NUMBER OF CORRECTION',5X,':',I5)
      END
      SUBROUTINE   MADSKY(X,N,K,AMINX,RHOLR)
C
C
C        FL = 0  IS A FUNCTION OF LAMDA, SEE EQ. 2.10 ON PAGE 13
C        FP = THE FIRST DERIVATIVE OF FL
C        TO FIND THE ROOT OF THIS FUNCTION WHICH LIES BETWEEN 0 AND
C        AMINX,THIS PROGRAM USES THE COMBINATION OF NEWTON AND BISECTION
C        METHODS WHICH WAS DEVELOPED BY MARC ( MATHEMATICAL ANALYSIS
C        RESEARCH CORPORATION,CLAREMONT,CA )
C
      DIMENSION CHISQ(2),X(100),N(100),RHOLR(2)
      DATA CHISQ/1.642,0.708/
C
C        EPS = IS THE SOLUTION TOLERANCE, ITERATION CEASES WHEN TWO
C        SUCCESSIVE APPROXIMATIONS DIFFER BY LESS THAN EPS
C
      EPS=AMINX*0.0001
      DO 225 L=1,2
      CONST=CHISQ(L)/2.0
C        SET THE FIRST APPROXIMATION HALFWAY BETWEEN 0 AND AMINX
C
      COLD=AMINX/2.0
  205 F=0.0
      FP=0.0
      DO 210 J=1,K
      F=F+FLOAT(N(J))*ALOG(1.0-COLD/FLOAT(N(J)))-X(J)*ALOG(1.0-COLD/X(J)
     a)
  210 FP=FP+X(J)/(X(J)-COLD)-FLOAT(N(J))/(FLOAT(N(J))-COLD)
      FL=F-CONST
      R=FL/FP
C        COLD = IS THE OLD APPROXIMATION
C        CNEW = IS THE NEW APPROXIMATION
      CNEW=COLD-R
C        WHENEVER THE NEW APPROXIMATION LARGER THAN AMINX
C        SET THE OLD APPROXIMATION HALFWAY BETWEEN THE OLD APPROXIMATION
C        AND AMINX TO FIND THE NEXT NEW APPROXIMATION,OTHERWISE USE
C        THE USUAL NEWTON METHOD
      IF(AMINX.LE.CNEW) GO TO 211
      IF(ABS(CNEW-COLD).LE.EPS) GO TO 215
      COLD=CNEW
      GO TO 205
  211 COLD=(COLD+AMINX)/2.0
      GO TO 205
  215 A=CNEW
      RHOLR(L)=1.0
      DO 220 I=1,K
  220 RHOLR(L)=RHOLR(L)*(X(I)-A)/(FLOAT(N(I))-A)
  225 CONTINUE
      RETURN
      END
      SUBROUTINE   RMMLI(X,N,K,PSI,RHOBIN,RHOMOD,Y,SIGMA2)
C
C
C        SIGMA2 = MAX.L'HOOD VARIANCE ESTIMATOR
C        HATN = PSEUDO SAMPLE SIZE
C        HATN*PSI = PSEUDO SUCCESS
C        MDBETI = SUBROUTINE NAME FOR COMPUTING THE INVERSE OF INCOMPLETE
C                 BETA FUNCTION
C        A = FIRST PARAMETER OF BETA FUNCTION
C        B = SECOND PARAMETER OF BETA FUNCTION
C        APLUS = FIRST PARAMETER OF BETA FUNCTION
C        BMIN = SECOND PARAMETER OF BETA FUNCTION
C        THESE LAST TWO PARAMETERS ARE TO BE USED IN RANDOMIZED
C        EASTERLING PROCEDURE
C
C
      DIMENSION X(100),N(100),RHOBIN(2),RHOMOD(2)
      RATIO=0.0
      DO 310 I=1,K
          RATIO = RATIO + (FLOAT(N(I))-X(I))/(FLOAT(N(I))*X(I))
  310 CONTINUE
      SIGMA2=PSI*PSI*RATIO
      HATN=PSI*(1.0-PSI)/SIGMA2
      NA = IFIX(HATN*PSI+1.0)
      NB = IFIX(HATN-HATN*PSI+1.0+1.0)
      A = FLOAT(NA)
      APLUS=A+Y
      B = FLOAT(NB)
      BMIN=B-Y
      P=0.1
      CALL MDBETI(P,A,B,Z,IER)
      CALL MDBETI(P,APLUS,BMIN,R,IER)
      RHOBIN(1) = Z
      RHOMOD(1)=R
      P=0.2
      CALL MDBETI(P,A,B,Z,IER)
      CALL MDBETI(P,APLUS,BMIN,R,IER)
      RHOBIN(2) = Z
      RHOMOD(2)=R
      RETURN
      END
      SUBROUTINE   AMLMAN(F,X,N,K,MINN,PSI,RHOAML,RHOMAN,IFLAG,SIGMA2)
C
C
      DIMENSION X(100),N(100)
      DIMENSION ZALPHA(2),RHOAML(2),RHOMAN(2),F(100)
      DATA ZALPHA/1.282,0.845/
C
C        COMPUTE LCL USING MAX. L'HOOD PROCEDURE
C
      DO 320 I = 1,2
           RHOAML(I) = PSI-ZALPHA(I)*SQRT(SIGMA2)
  320 CONTINUE
      AMINN = FLOAT(MINN)
      KK = 0
      AM = 0.0
      AM1 = 0.0
C        IF UNEQUAL SAMPLE SIZE GO TO 326
      IF (IFLAG.EQ.1) GO TO 326
      DO 325 I = 1,K
C
C        GO TO 325 : IGNORE ZERO-FAILURE COMPONENT
C
           IF (F(I).EQ.0.0) GO TO 325
           AM = AM + 1.0/FLOAT(N(I))
  325 CONTINUE
      AM = AMINN*AM
      GO TO 365
  326 CONTINUE
      DO 360 J = 1,K
C
C        GO TO 350 : IGNORE ALL ZERO-FAILURE COMPONENTS WITH SAMPLE
C                    SIZE MINN,EXCEPT ONE
C
           IF ((F(J).EQ.0.0).AND.(MINN.EQ.N(J))) GO TO 350
           IF(MINN.EQ.N(J)) GO TO 330
           GO TO 340
C
C        ACCUMULATE ALL COMPONENTS THAT HAVE SAMPLE SIZE MINN WHICH
C        EXHIBIT FAILURE
C
  330      KK = KK + 1
           GO TO 360
C
C        IGNORE ZERO-FAILURE COMPONENT IF THE CONDITION IS TRUE
C
  340      IF (F(J).EQ.0.0) GO TO 360
           AM = AM + 1.0/FLOAT(N(J))
           GO TO 360
  350      AM1 = 1.0/FLOAT(N(J))
  360 CONTINUE
```

```
C          IF THE CONDITION IS TRUE COMPUTE AM BY IGNORING ALL COMPONENTS
C          THAT HAVE SAMPLE SIZE MINN WHICH EXHIBIT NO FAILURE,OTHERWISE
C          COMPUTE AM BY IGNORING ALL COMPONENTS THAT HAVE SAMPLE SIZE MINN
C          WHICH EXHIBIT NO FAILURE,EXCEPT ONE
C
      IF (KK.GT.0) GO TO 361
      AM = AMINN*(AM + AM1)
      GO TO 365
  361 AM=AMINN*(AM+FLOAT(KK)*(1.0/AMINN))
  365 CONTINUE
      PHAT = 1.0-FSI
      ANOT = AMINN*(1.0-0.5*PHAT*PHAT)*(1.0-0.5*PHAT)
      AMS = (0.5*(1.0+1.0/AM))/ANOT + PHAT/(1.0-0.5*PHAT)
      VS = (0.5*(1.0+1.0/AM)*AMS)/ANOT
      C = 1.0-VS/((3.0*AMS)*(3.0*AMS))
      D = SQRT(VS)/(3.0*AMS)
C
C          COMPUTE LCL USING MANN'S FORMULA
C
      DO 370 I = 1,2
         CONST = C + D*ZALPHA(I)
         RHOMAN(I) = EXP(-AMS*(CONST*CONST*CONST))
  370 CONTINUE
      RETURN
      END
      SUBROUTINE WOODBG(N,F,RHOGAM,K,MAX)
C
C
      DIMENSION N(100),F(100),RHOGAM(2),CHI90(120),CHI80(120),Q(100)
C
      DATA CHI90/0.0158,0.211,0.584,1.064,1.610,2.204,2.833,3.490,
C
     1           4.168,4.865,5.578,6.304,7.042,7.790,8.547,9.312,
C
     2           10.085,10.865,11.651,12.443,13.240,14.041,14.848,
C
     3           15.659,16.473,17.292,18.114,18.939,19.768,20.599,
C
     4           21.443,22.270,23.110,23.952,24.796,25.643,26.492,
C
     5           27.342,28.195,29.050,29.907,30.765,31.625,32.487,
C
     6           33.350,34.215,35.081,35.949,36.818,37.688/
C
      DATA CHI80/0.0642,0.446,1.005,1.649,[illegible],3.070,3.822,4.594,
C
     1           5.380,6.179,6.989,7.807,8.634,9.467,10.307,11.152,
C
     2           12.002,12.857,13.716,14.578,15.445,16.314,17.187,
C
     3           18.062,18.940,19.820,20.703,21.588,22.475,23.364,
C
     4           24.255,25.147,26.042,26.938,27.835,28.735,29.635,
C
     5           30.537,31.440,32.345,33.250,34.157,35.063,35.974,
C
     6           36.884,37.795,38.707,39.620,40.534,41.449/
C
      DATA Z1,Z2/1.282,0.845/
C
      SHSTAR = 0.0
      SUM = 0.0
      DO 430 J = 1,K
         AN = FLOAT(N(J))
         A = (2.0*AN-3.0)/(2.0*(AN-1.0))
         B = 0.5*(AN/(AN-1.0))
         Q(J) = F(J)/AN
         T = A*Q(J) + B*Q(J)*Q(J)
         RT = T/AN
         IF (MAX.EQ.J) GO TO 410
         GO TO 420
C     INTRODUCE ONE MORE FAILURE TO THE COMPONENT WITH LARGEST
C     SAMPLE SIZE OR TO THE FIRST COMPONENT IF ALL SAMPLE SIZES
C     ARE EQUAL
  410    Q(J) = (F(J)+1.0)/AN
```

```
           THAT = A*C(J) + B*C(J)*Q(J)
           T = 0.5*(THAT + T)
           RT = T/AN
  420      SHSTAR = SHSTAR + T
           SUM = SUM + RT
  430 CONTINUE
      RSTAR = (SHSTAR*SHSTAR)/SUM
      NRSTAR = IFIX(2.0*RSTAR+1.0)
      IF (NRSTAR.LE.50) GO TO 440
C
C        IF DEGREES OF FREEDOM EXCEED 50 COMPUTE THE CHI-SQUARE PERCENTILE
C        USING THIS FORMULA
C
      C1 = SQRT(2.0*FLOAT(NRSTAR)-1.0)-Z1
      C2 = SQRT(2.0*FLOAT(NRSTAR)-1.0)-Z2
      CHI90(NRSTAR) = 0.5*C1*C1
      CHI80(NRSTAR) = 0.5*C2*C2
  440 CONTINUE
C
C        COMPUTE THE LCL USING LOG-GAMMA FORMULA
C
      RHOGAM(1) = EXP(-SHSTAR*FLOAT(NRSTAR)/CHI90(NRSTAR))
      RHOGAM(2) = EXP(-SHSTAR*FLOAT(NRSTAR)/CHI80(NRSTAR))
      RETURN
      END
      SUBROUTINE CMPARE(Z,STDDEV,CTILE,INDEX,ZBAR,RS)
C
C
C        THIS SUBROUTINE COMPUTE THE 90-TH AND 80-TH PERCENTILES,
C        MEAN,STANDARD DEVIATION,AND PERCENTILE POINTS CLOSEST TO RS
C
C
      DIMENSION Z(2,500),STDDEV(2),CTILE(2),INDEX(2),ZBAR(2),Y(2,500),
     *          Z1(500),Z2(500)
      DO 10 J=1,500
           Z1(J)=Z(1,J)
           Z2(J)=Z(2,J)
   10 CONTINUE
C
C        SUBROUTINE FXSORT IS USED TO ORDER THE LDL
C
      CALL FXSORT(Z1,1,500)
      CALL FXSORT(Z2,1,500)
      CTILE(1)=Z1(450)
      CTILE(2)=Z2(400)
      DO 20 J=1,500
           Z(1,J)=Z1(J)
           Z(2,J)=Z2(J)
   20 CONTINUE
      DO 45 I=1,2
           SUM=0.0
           DO 30 J=1,500
                SUM=SUM+Z(I,J)
                Y(I,J)=ABS(RS-Z(I,J))
   30      CONTINUE
           ZBAR(I)=SUM/500.0
           SUM=0.0
           DO 35 J=1,500
                SUM=SUM+(Z(I,J)-ZBAR(I))*(Z(I,J)-ZBAR(I))
   35      CONTINUE
           STDDEV(I)=SQRT(SUM/499.0)
           AMINY=Y(I,1)
           DO 40 J=2,500
           IF(Y(I,J).LT.AMINY) GO TO 36
           GO TO 40
   36      AMINY=Y(I,J)
           A=FLOAT(J)/5.0
   40      CONTINUE
           INDEX(I)=IFIX(A)
   45 CONTINUE
      RETURN
      END
```

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